Evaluation Of Bit Error Rate Performance Of Multi-Level Differential Phase Shift Keying (M-ARY DPSK)

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***Abstract*— In this paper, the evaluation of bit error rate performance of Multi-Level Differential Phase Shift Keying (M-ARY DPSK) is studied. Relevant analytical equations for the computation of the BER of M-ARY DPSK are presented and some sample numerical computations were performed in respect of the BER performance. Particularly, a normalized bit error rate (BER) performance analysis was presented and the results clearly showed the dynamics of the BER performance of M-ARY DPSK with respect to energy per bit to noise power density ratio (Eb/No) and modulation index (M). The normalization was conducted with respect to a reference BER of 1.395E-01, which is the BER for M=4 and Eb/No = 0 dB. The results showed that the BER performance of M-ARY DPSK for M = 256 was better than that of M = 32 for Eb/No < 8 dB . Also, the BER performance of M-ARY DPSK for M = 4 was better than that of M = 32 for Eb/No ≥ 0. dB and better than that of M = 256 for Eb/No > 0.5 dB. In essence, the BER performance of M-ARY DPSK depends on the Eb/No value as well. In some case, both better BER and M value can be achieved with the M-ARY DPSK modulation. Essentially, the major research finding in this paper is that for the M-ARY DPSK modulation, proper selection of the M value with respect to the prevailing value of Eb/No is essential for better error performance and throughout.**

Keywords*— *Bit Error Rate (BER), Differential Phase Shift Keying (M-ARY DPSK), Modulation, Normalized BER, Energy Per Bit To Noise Power Density Ratio (Eb/No), Modulation Index

I. INTRODUCTION

Differential Phase Shift Keying (DPSK) is a category of phase modulation scheme used to transmit data by changing the carrier signal's phase [1,2,3,4,5,6,7,8]. However, unlike the classical Phase Shift Keying (PSK) modulation scheme, in the DPSK scheme, the modulated signal phase is shifted or changed relative to the value of the previous signal element. Rather, the modulated signal phases follow the low state or the high state of the previous signal element.

One implementation, in the case of Differential Binary Phase Shift Keying (DBPSK), can be such that if the current data bit is Low (that is '0'), then the signal phase is not reversed (that is, the phase change is 0 °) and the signal continued unchanged [9,10,11,12,13,14,15,16]. However, if current data bit is High (that is '1'), then the signal phase is reversed (that is, the phase change is 180 °). Another implementation, in the case of Differential Quadrature Phase Shift keying (DQPSK), can be such that the phase-shifts of the signal relative to the value of the previous signal element are $0°$, $90°$, $180°$, $-90°$ which corresponds to data values of "00", "01", "11", and "10" respectively [17,18,19,20,21,22,23,24]. Notably, in the case of DBPSK, the number of different phases (denoted as M) is 2 while the number of digital data bit (denoted as n) used if 1. Again, in the case of DQPSK, the number of different phases is 4 (that is, M = 4) while the number of digital data bit 2 (that is, n = 2). Accordingly, in Multi-Level Differential Phase Shift Keying (M-ARY DPSK) modulation scheme, the number of different phases is M while the number of digital data bit n is $\mathrm{Log}_2(M)$ [25,26,27,28].

Generally, DPSK is considered as improved version of the classical PSK modulation technique. In comparison with the PSK modulation, the DPSK modulation schemes are less susceptible to interferences and noise and they offer higher $\frac{\varepsilon_b}{N_0}$ (which is ratio of energy per bit to spectral noise density) and higher spectrally efficient. Moreover, in M-ARY DPSK modulation schemes there is no need for a reference signal or reference oscillator. As such, M-ARY DPSK demodulators do not require synchronous (coherent) carrier or synchronization of the transmitter and the receiver.

In this paper, the evaluation of bit error rate (BER) performance of Multi-Level Differential Phase Shift Keying (M-ARY DPSK) is presented. The BER performance of the M-ARY DPSK modulation for different values of modulation order (M) is presented based on a generalized BER formula and some numerical examples.

II. ERROR PERFORMANCE OF M-ARY DPSK MODULATION

A. Bit Error Probability (BER) of M-ary DPSK Modulation

The **Multi-Level Differential Phase Shift Keying (M-ARY** DPSK) bit error probability **(BER)** in terms of Q function is expressed as;

$$P_{bM-DPSK}(Q) = \left(\frac{2}{\mathrm{Log}_2(M)}\right) Q\left(Sin\left(\frac{\pi}{2(M)}\right)\sqrt{\left(4(\mathrm{Log}_2(M))\left(\frac{\varepsilon_b}{N_0}\right)\right)}\right) M \geq 4 \quad (1)$$

Where the modulation index is denoted as M, the energy per bit is denoted as the ε_b and the noise power density denoted as N_0.

When the complementary error function (erfc) is used, the **M-ARY** DPSK bit error probability **(BER)** can be expressed as;

$$P_{bM-DPSK}(erfc) = \left(\frac{1}{\mathrm{Log}_2(M)}\right) erfc\left(Sin\left(\frac{\pi}{2(M)}\right)\sqrt{\left(2(\mathrm{Log}_2(M))\left(\frac{\varepsilon_b}{N_0}\right)\right)}\right) \; for\; M \geq 4 \quad (2)$$

Similarly, when the error function (erf) is used, the **M-ARY** DPSK bit error probability **(BER)** can be expressed as;

$$P_{bM-DPSK}(erf) = \left(\frac{1}{\mathrm{Log}_2(M)}\right)\left(1 - erfc\left(Sin\left(\frac{\pi}{2(M)}\right)\sqrt{\left(2(\mathrm{Log}_2(M))\left(\frac{\varepsilon_b}{N_0}\right)\right)}\right)\right) \; for\; M \geq 4 \quad (3)$$

B. Symbol Error Probability (BER) of M-ary DPSK Modulation

The **Multi-Level Differential Phase Shift Keying (M-ARY** DPSK) symbol error probability **(SER)** in terms of Q-function is expressed as;

$$P_{sM-DPSK}(Q) = (2)Q\left(Sin\left(\frac{\pi}{2(M)}\right)\sqrt{\left(4(\mathrm{Log}_2(M))\left(\frac{\varepsilon_b}{N_0}\right)\right)}\right) M \geq 4 \quad (4)$$

When the complementary error function (erfc) is used, the **M-ARY** DPSK symbol error probability **(SER)** can be expressed as;

$$P_{sM-DPSK}(erfc) = erfc\left(Sin\left(\frac{\pi}{2(M)}\right)\sqrt{\left(2(\mathrm{Log}_2(M))\left(\frac{\varepsilon_b}{N_0}\right)\right)}\right) \; for\; M \geq 4 \quad (5)$$

Similarly, when the error function (erf) is used, the **M-ARY** DPSK bit error probability **(SER)** can be expressed as;

$$P_{sM-DPSK}(erf) = \left(1 - erfc\left(Sin\left(\frac{\pi}{2(M)}\right)\sqrt{\left(2(\mathrm{Log}_2(M))\left(\frac{\varepsilon_b}{N_0}\right)\right)}\right)\right) \; for\; M \geq 4 \quad (6)$$

C. Multi-Level Differential Phase Shift Keying (M-ARY DPSK) Generalized BER Formula

For M = 2, the BER of **M-ARY** DPSK is given as;

$$P_{bDBPSK} = \left(\frac{1}{2}\right)\left(e^{-\left(\frac{\varepsilon_b}{N_0}\right)}\right) M \geq 4 \quad (7)$$

However, for $M \geq 4$, a generalized BER of **M-ARY** DPSK modulation is expressed in terms of $erfc$ can be given as shown in Eq 9;

$$P_{bM-DPSK}(erfc) = \left(\frac{1}{\mathrm{Log}_2(M)}\right) erfc\left(\left(\left(Sin\left(\frac{\pi}{2(M)}\right)\right)\sqrt{(2)\mathrm{Log}_2(M)}\right)\sqrt{\left(\frac{\varepsilon_b}{N_0}\right)}\right) \; for\; M \geq 4 \quad (8)$$

$$P_{bM-DPSK}(erfc) = (A)erfc\left(B\sqrt{\left(\frac{\varepsilon_b}{N_0}\right)}\right) \quad (9)$$

Where

$$A = \frac{1}{\mathrm{Log}_2(M)} \quad (10)$$

$$B = \left(Sin\left(\frac{\pi}{2(M)}\right)\right)\sqrt{(2)\mathrm{Log}_2(M)} \quad (11)$$

III. RESULTS AND DISCUSSION

The BER performance of the M-Ary DPSK modulation is examined with some numerical examples based on the generalized BER formula. The numerical results in Table 1 show the parameter values for A and B as specified in the generalized BER formula for modulation index (M) values of 2 to 256. The results in Table 2 and Figure 1 show the BER values for the M-ARY DPSK) modulation for various values of Eb/No (dB) and M. Similarly, results in Table 3 and Figure 2 show the normalized BER versus Eb/No (dB) for M-ARY DPSK modulation for M =4, 32 and 256 where the normalization BER is 1.395E-01 which is the BER For M=4 and Eb/No = 0 dB.

The results in Table 2 and Figure 1 show that for any given Eb/No the BER decreases as M decreases. In essence, for any given Eb/No , the BER for M =4 is smaller and hence better than the BER for M = 32. It also implies that, for any given Eb/No, the BER for M =32 is smaller and hence better than the BER for M = 256. However, the result of the normalized BER gives a clearer picture of the dynamics of the BER performance of **M-ARY DPSK modulation.**

Table 1 The Values of A and B in the M-Ary DPSK modulation BER generalized formula

$P_{bM-DPSK}(erfc) = (A)erfc\left(B\sqrt{\left(\frac{\varepsilon_b}{N_0}\right)}\right)$		$A = \frac{1}{\text{Log}_2(M)}$	$B = \left(Sin\left(\frac{\pi}{2(M)}\right)\right)\sqrt{(2)\text{Log}_2(M)}$	Alternative Form of A and B	
M	$\text{Log}_2(\text{M})$	**A**	**B**	**A**	**B**
2	1	1/2	1	0.5	1
4	2	1/2	137/179	0.5	0.765366865
8	3	1/3	54/113	0.333333333	0.477871743
16	4	1/4	28/101	0.25	0.277234338
32	5	1/5	9/58	0.2	0.15516561
64	6	1/6	59/694	0.166666667	0.085013309
128	7	1/7	9/196	0.142857143	0.045915892
256	8	1/8	4/163	0.125	0.024543539

Table 2 The Multi-Level Differential Phase Shift Keying (M-ARY DPSK) modulation BER versus Eb/No (dB)

Modulation index, M	2	4	8	16	32	64	128	256
bits/symbol	1	2	3	4	5	6	7	8
Eb/No(dB)	**M-DPSK BER For M=2**	**M-DPSK BER For M=4**	**M-DPSK BER For M=8**	**M-DPSK BER For M=16**	**M-DPSK BER For M=32**	**M-DPSK BER For M=64**	**M-DPSK BER For M=128**	**M-DPSK BER For M=256**
0	1.839E-01	1.395E-01	1.664E-01	1.738E-01	1.653E-01	1.507E-01	1.355E-01	1.215E-01
1	1.318E-01	1.057E-01	1.450E-01	1.627E-01	1.600E-01	1.483E-01	1.343E-01	1.210E-01
3	8.446E-02	7.445E-02	1.225E-01	1.503E-01	1.540E-01	1.454E-01	1.330E-01	1.204E-01
4	4.668E-02	4.778E-02	9.934E-02	1.365E-01	1.471E-01	1.422E-01	1.315E-01	1.197E-01
5	2.116E-02	2.713E-02	7.648E-02	1.214E-01	1.393E-01	1.385E-01	1.297E-01	1.188E-01
6	7.372E-03	1.312E-02	5.507E-02	1.052E-01	1.305E-01	1.342E-01	1.277E-01	1.179E-01
8	1.806E-03	5.133E-03	3.634E-02	8.813E-02	1.206E-01	1.293E-01	1.254E-01	1.168E-01
9	2.768E-04	1.518E-03	2.141E-02	7.075E-02	1.096E-01	1.237E-01	1.227E-01	1.155E-01
10	2.270E-05	3.099E-04	1.086E-02	5.376E-02	9.755E-02	1.173E-01	1.196E-01	1.141E-01
11	8.083E-07	3.865E-05	4.530E-03	3.806E-02	8.459E-02	1.101E-01	1.161E-01	1.124E-01
13	9.462E-09	2.505E-06	1.458E-03	2.457E-02	7.096E-02	1.020E-01	1.120E-01	1.105E-01
14	2.513E-11	6.788E-08	3.328E-04	1.406E-02	5.705E-02	9.304E-02	1.074E-01	1.082E-01
15	9.234E-15	5.762E-10	4.816E-05	6.868E-03	4.344E-02	8.316E-02	1.021E-01	1.057E-01

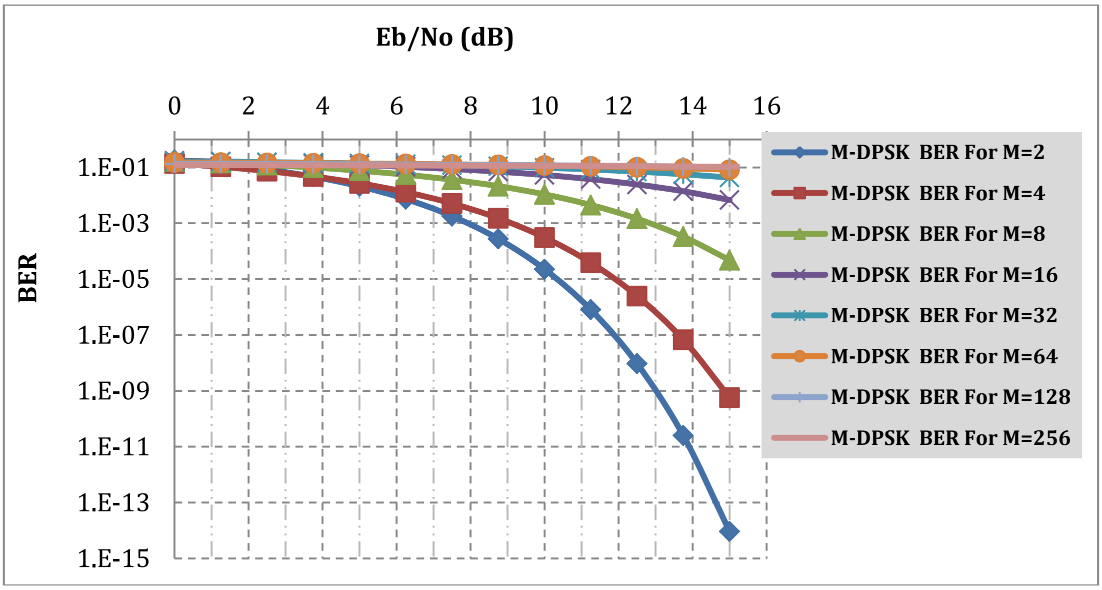

Figure 1 The Multi-Level Differential Phase Shift Keying (M-ARY DPSK) modulation BER versus Eb/No (dB)

A normalized BER is examined in respect of the results in Table 2 where the normalized BER is given as;

$$\text{Normalized BER} = \frac{BER}{\text{References BER}} \quad (12)$$

The results for the normalized BER for M =4, M =32 and M =256 are shown in Table 4 and Figure 2. The normalization can be conducted with respect to any reference BER. However, in case of the results in Table 3 and Figure 2, the reference BER is 1.395E-01, which is the BER for M=4 and Eb/No = 0 dB. The results shows that the BER performance of M-ARY DPSK for M = 256 is better than that of M = 32 for Eb/No < 8 dB . Also, the BER performance of M-ARY DPSK for M = 4 is better than that of M = 32 for Eb/No ≥ 0. dB and better than that of M = 256 for Eb/No > 0.5 dB. The importance of the results in Figure 2 and table 3 is that they show the dynamics of the BER performance of the M-ARY DPSK modulation for different values of M.

Table 3 Normalized BER versus Eb/No (dB) For (M-ARY DPSK) modulation (Normalized with respect to 1.395E-01 which is the BER For M=4 and Eb/No = 0 dB)

Signal Levels or Modulation Order, M	4	32	256
K bits/symbol	2	5	8
Eb/No(dB)	M-DPSK **Normalized** BER For M=4	M-DPSK **Normalized** BER For M=32	M-DPSK **Normalized** BER For M=256
0	1.000E+00	1.184E+00	8.710E-01
1	7.572E-01	1.147E+00	8.672E-01
3	5.336E-01	1.103E+00	8.627E-01
4	3.424E-01	1.054E+00	8.576E-01
5	1.944E-01	9.981E-01	8.517E-01
6	9.400E-02	9.349E-01	8.449E-01
8	3.678E-02	8.640E-01	8.370E-01
9	1.088E-02	7.853E-01	8.280E-01
10	2.221E-03	6.991E-01	8.175E-01
11	2.770E-04	6.062E-01	8.055E-01
13	1.795E-05	5.085E-01	7.916E-01
14	4.864E-07	4.089E-01	7.756E-01
15	4.129E-09	3.113E-01	7.572E-01

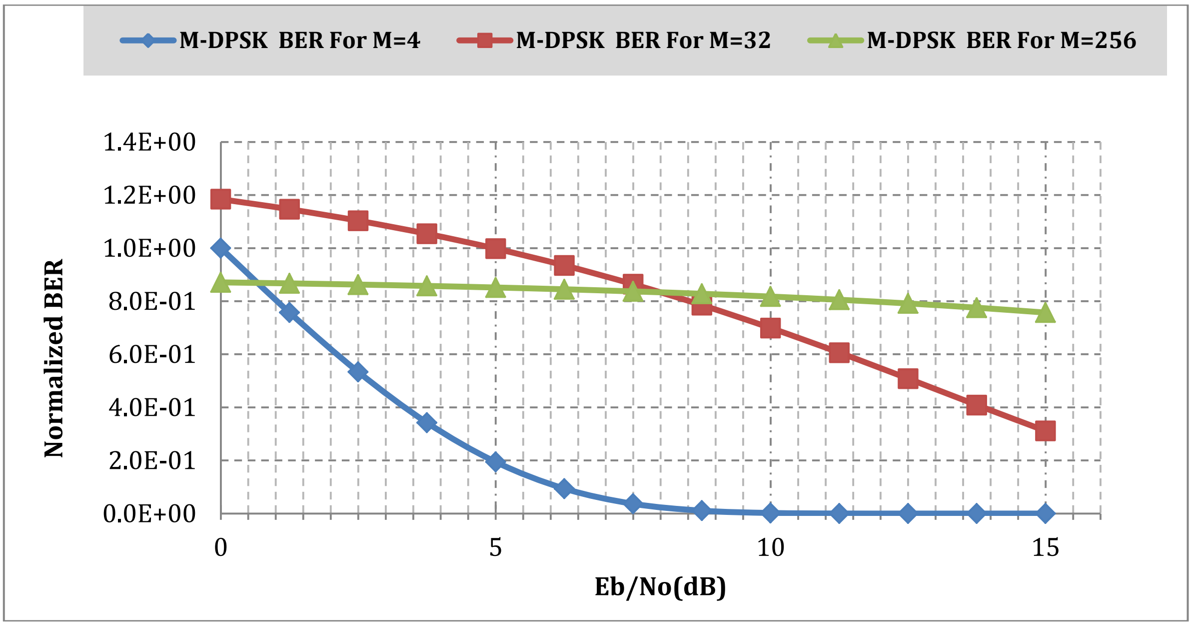

Figure 2 Normalized BER versus Eb/No (dB) For (M-ARY DPSK) modulation (Normalized with respect to 1.395E-01 which is the BER For M=4 and Eb/No = 0 dB)

4.0 CONCLUSION

The multi-level Differential Phase Shift Keying (M-ARY DPSK) is studied and the BER performance is evaluated for different values of modulation index. Particularly, a normalized bit error rate (BER) performance analysis was presented and the results clearly showed the dynamics of the BER performance of M-ARY DPSK with respect to energy per bit to noise power density ratio (Eb/No) and modulation index. The results showed that the BER performance of M-ARY DPSK depends on the Eb/No value as well. In some case, both better BER and M value can be achieved with the M-ARY DPSK modulation when the prevailing value of Eb/No is known and the appropriate value of M is selected for the M-ARY DPSK modulation. Essentially, the major research finding in this paper is that for the M-ARY DPSK modulation, proper selection of the M value with respect to the prevailing value of Eb/No is essential for better error performance and throughout.

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