# ENHANCED BISECTION ITERATION METHOD APPLIED IN FADE MARGIN-BASED OPTIMAL PATH LENGTH FOR FIXED POINT TERRESTRIAL MICROWAVE COMMUNICATION LINK WITH KNIFE EDGE DIFFRACTION LOSS

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Abstract- In this paper, the development and evaluation of enhanced bisection iteration method applied in fade margin-based optimal path length for fixed point terrestrial microwave communication link with knife edge diffraction loss is presented. The relevant mathematical expressions for the computation of the optimal path length were presented along with the enhanced bisection iteration algorithm. Sample numerical microwave link was used to compare the convergence cycle of the two algorithms and to study the impact of frequency on the performance of the algorithms. The results show that for the given microwave link, the classical Bisection method has convergence cycle of 17 to 13 as the frequency is varied from 10 GHz to 200GHz. On the other hand, the convergence cycle for the Enhanced Bisection algorithm varied from 5 to 8 cycles for all frequencies. In essence, the Enhanced Bisection method developed in this paper has proven to be the better algorithm for determination of the optimal path length of terrestrial microwave link.

Keywords— Bisection Method, Fade Margin, Optimal Path Length, Iteration Method , Terrestrial Microwave, Communication Link, Knife Edge Diffraction Loss

# I. INTRODUCTION

Terrestrial microwave line of sight (LoS) communication link [1] requires that the transmitting antenna and the receiving antenna be located on high building roofs or high towers, so as to avoid obstacles on the LoS propagation path. As microwave signals are being propagated from the transmitter antenna to the receiver's antenna, it is observed that they travel in a straight line except when they are reflected or refracted by objects [2,3,4,5]. Also, as the signals propagate farther away from the radiating antenna, the signals spread away from the line of sight path. Further explanation has it that the Fresnel zone is an ellipsoid boundary, inside which most of the signal powers reaches the receiving antenna. Any attempt by any object to interfere within the first Fresnel zone leads to a very serious phenomenon called attenuation [6,7].

Attenuation or signal path loss is the reduction in power density of an electromagnetic wave as it propagates through a medium [8,9,10]. Signal path loss can occur due to environmental factors. In terrestrial LoS microwave communication link design, the maximum path length depends, among other things, on the path loss (which can be modeled as free space path loss, (FSPL) and the maximum fade depth determined from the link parameters. In practice, mostly rain and multipath fading are considered and they are taken to be mutually exclusive when determining the fade depth for terrestrial LoS microwave communication links [11]. As such, the maximum fade depth is taken to be rain fading or multipath fading; whichever one is larger. Furthermore, for any given set of terrestrial LoS microwave communication link parameters and specified fade margin  $(f_{ms})$ , the maximum path length determined from the FSPL loss (d<sub>mfsp</sub>) and the maximum path length determined from the specified fade margin  $(d_{msfm})$  may differ [12,13].

This paper focus on the development and evaluation of enhanced bisection method for computing the optimal path length  $(d_{mop})$  for terrestrial line of sight microwave communication link. Specifically, the optimal path length is that path length at which the system operating margin (or fade margin) is just satisfied and the path length determined from FSPL loss is the same as the path length determined from the maximum fade depth. In this paper, sample numerical example is used to evaluate the performance of the enhanced bisection iteration method and also to study the effect of frequency on the performance of the algorithms.

## II. METHODOLOGY

## A. Link Budget Equation Including Single Knife Edge Diffraction Loss

The maximum allowable single knife edge diffraction loss expected in a terrestrial microwave link can be specified in terms of maximum line of sight (LOS) percentage clearance,  $P_c$  [14,15]. In terms of  $P_c$ , the Fresnel-Kirchhoff diffraction parameter, V is given as [16];

$$V = \left(\frac{(\sqrt{2})P_c}{100}\right) \tag{1}$$

Lee's approximation model for computing single knife edge diffraction loss,  $G_d$  with respect to V is given as follows [16,17,18,19,20];

Lee's approximation for single knife edge diffraction loss, 
$$G_d$$
 as a function of LOS percentage clearance,  $P_c$  is given as [16];

$$\begin{array}{l}
G_{d} = 20 \log \left( \frac{0.225}{v} \right) & \text{for } V > 2.4 \\
\begin{cases}
G_{d} = 0 & \text{for } P_{c} < -70.7107\% \\
G_{d} = 20 \log \left( 0.5 - 0.62 \left( \frac{P_{c}}{70.71068} \right) \right) & \text{for } -70.7107\% \leq P_{c} \leq 0\% \\
G_{d} = 20 \log \left( 0.5 \exp(-0.95 \left( \frac{P_{c}}{70.71068} \right) \right) & \text{for } 0\% \leq P_{c} \leq 70.7107\% \\
\begin{cases}
G_{d} = 20 \log \left( 0.4 - \sqrt{0.1184 - \left( 0.38 - 0.1 \left( \frac{P_{c}}{70.71068} \right) \right)^{2}} \right) & \text{for } 70.7107\% \leq P_{c} \leq 169.7056\% \\
\end{cases}$$

$$\begin{array}{c}
G_{d} = 20 \log \left( \frac{0.225}{\left( \frac{P_{c}}{70.71068} \right)} \right) & \text{for } P_{c} > 169.7056\% \\
\end{array}$$

$$\begin{array}{c}
(3) \\
G_{d} = 20 \log \left( \frac{0.225}{\left( \frac{P_{c}}{70.71068} \right)} \right) & \text{for } P_{c} > 169.7056\% \\
\end{array}$$

$$\begin{array}{ll} G_{d} &= 0 & \text{for } P_{c} < -70.7107\% \\ G_{d} &= 20 \log \left( 0.5 - \left( \frac{P_{c}}{114.0495} \right) \right) & \text{for } -70.7107\% \leq P_{c} \leq 0\% \\ G_{d} &= 20 \log \left( 0.5 \exp \left( - \left( \frac{P_{c}}{74.43229276} \right) \right) & \text{for } 0\% \leq P_{c} \leq 70.7107\% \\ \end{array} \right) \\ &= 20 \log \left( 0.4 - \sqrt{0.1184 - \left( 0.38 - \left( \frac{P_{c}}{707.1067812} \right) \right)^{2}} \right) & \text{for } 70.7107\% \leq P_{c} \leq 169.7056\% \\ G_{d} &= 20 \log \left( \frac{0.003181981}{P_{c}} \right) & \text{for } P_{c} > 169.7056\% \end{array} \right)$$

Link budget equation that includes single knife edge diffraction loss,  $G_d$  is given as follows:

for V < -1

for  $-1 \le V \le 0$ 

for  $0 \leq V \leq 1$ 

V > 2.4

(2)

Received Power = Transmitted Power + Sum of Gains - Sum of Losses (5)

$$P_R = P_T + (G_T + G_R) - (LFSP + G_d + L_T + L_M + L_R)$$
 (6) where:

 $P_R$  = Received Signal Power (dBm)

 $G_d$ 

 $G_d = 0$ 

 $G_d = 20\log(0.5 - 0.62v)$ 

 $G_d = 20\log(0.5\exp(-0.95v))$ 

 $G_d = 20\log(0.4 - \sqrt{0.1184 - (0.38 - 0.1v)^2})$  for  $1 \le V \le 2.4$ 

 $P_{T}$  = Transmitter Power Output (dBm)

G<sub>T</sub> = Transmitter Antenna Gain (dBi)

 $G_R$  = Receiver Antenna Gain (dBi)

LFSP = Free Space Path Loss (dB).

 $G_d$  = Single knife edge diffraction loss(dB).

 $L_T$  = Losses from Transmitter (cable, connectors etc.) (dB)

 $L_R$  = Losses from Receiver (cable, connectors etc.) (dB)

 $L_M$  = Misc. Losses (fade margin, polarization misalignment etc.) (dB)

The received signal strength is given as;

$$P_{\rm R} = P_{\rm T} + G_{\rm T} + G_{\rm R} - LFSP - G_d \quad (7)$$
  
Hence,

(8)LFSP =  $P_T + G_T + G_R - P_R - G_d = 32.4 + 20 \log(f^*1000) + 20 \log(d)$ 

where

- f frequency of the emitted signal in GHz
- length of the link in km d

Therefore, with respect to the Free Space Path Loss, the effective path length  $(d_e)$  is given as:

$$d_e = 10^{\left(\frac{(P_T + G_T + G_R - fm_s - G_d - P_s) - 32.4 - 20\log(f*1000)}{20}\right)} (9)$$

With respect to  $d_e$ , the Effective Free Space Path Loss, (LFSP<sub>e</sub>) is given as:

$$LFSP_{e} = 32.4 + 20 \log(f^{*}1000) + 20 \log(d_{e}) \quad (10)$$

*Effective Received Power*  $(P_{Re})$  is given as:

$$P_{Re} = P_T + G_T + G_R - LFSP_e - G_d$$
(11)

*Effective Fade Margin*  $(fm_e)$  is given as:

$$fm_e = (P_T + G_T + G_R) - G_d - (32.4 + 20 \log(f * 1000) + 20 \log(d_e)) - P_S$$
(12)

For microwave frequencies from 10 GHz and above, the rain fading is the dominant fade mechanism. The rain fade

depth ( $fd_{me}$ ) at a path length ( $d_e$ ) is given as [21];

$$fd_{me} = \max\left(\left(K_{v}(R_{po})^{\alpha_{v}}\right) * d_{e} , \left(K_{h}(R_{po})^{\alpha_{h}}\right) * d_{e}\right)\right) (13)$$
  
where:

are frequency dependent coefficients for  $k_h, \alpha_h$ horizontal polarization. They are given in (ITU\_R, 2005a)

kv,  $\alpha_{v}$  are frequency dependent coefficients for vertical polarization. They are given in (ITU\_R, 2005a)

 $\langle \gamma_{R_{po}} \rangle_h$  is the rain attenuation per kilometer for horizontal polarization

 $\langle \gamma_{R_{po}} \rangle_{v}$  is the rain attenuation per kilometer for horizontal polarization

(4)

*po* is the Percentage outage time (or Percentage unavailability time) of the link.

pa is the Percentage availability time of the link.

$$po = (100\% - pa)$$
 (14)

B. The Concept Of Optimal Path Length

The concept of optimal path length is captured in Figure 1. Based on the fade margin  $(fm_e)$ , the optimal path length  $(d_{mop})$  is obtain when the effective fade margin  $(fm_e)$  is equal to the effective fade depth  $(fd_{me})$ . Hence;

$$fm_e = fd_{me} \quad (15)$$

Now, when  $fm_e \neq fd_{me}$  there are two possible values for effective path length which can be regarded as  $d_{e1}$  and  $d_{e2}$  where  $d_{e1}$  is obtained when  $fm_s = fm_e$  in the determination of  $d_e$  as follows [11,12,13];

Similarly,  $d_{e2}$  is obtained when  $fm_s = fd_{me}$  in the determination of  $d_e$  as follows;

$$d_{e2} = 10^{\left(\frac{(P_{T} + G_{T} + G_{R} - f d_{me} - G_{d} - P_{S}) - 32.4 - 20 \log(f*1000)}{20}\right)}$$
(17)

Then, the optimal path length  $(d_{mop})$  is a between the two values,  $d_{e1}$  and  $d_{e2}$ . The optimal path length  $(d_{mop})$  will occur when  $fm_s = fd_{me}$  and hence  $d_{e1} = d_{e2}$ . In practice it may require long iterative process before the condition  $fm_s = fd_{me}$  can be attained. As such a prespecified relative error tolerance,  $\in_s$  is specified, such that, when  $fm_s = fd_{me} \leq \epsilon_s$  the value of  $d_{e1}$  is approximately equally to  $d_{e2}$ . Hence, the condition for the termination of the fade margin-based iteration is;

$$|fm_e - fd_{me}| \le |\epsilon_s| \tag{18}$$



where  

$$fd_{me} = max((K_v(R_{po})^{\alpha_v})*d_{e},(K_h(R_{po})^{\alpha_h})*d_e))$$
  
 $fm_e = (P_T + G_T + G_R) - G_d - (32.4 + 20log(f*1000) + 20log(d_e))$ 

Figure 1 The concept of optimal path length

#### III. THE CONCEPT OF BISECTION METHOD AND ENHANCED BISECTION METHOD

 $d_{eop} = \frac{d_{e2} - d_{e1}}{\beta} = \frac{d_{e2} - d_{e1}}{2}$ (20)

A. The Concept of Path Length–Based Bisection Iteration Method For The Determination Of Optimal Path Length

Given two initial path lengths  $(d_{e1} \text{ and } d_{e2})$  and if  $d_{e1} < d_{e2}$ , the optimal effective path length  $(d_{eop})$  can be computed using the bisection method [22,23,24] as follows;

$$d_{eop} = \frac{d_{e2} - d_{e1}}{\beta} \tag{19}$$

For the classic bisection method  $\beta = 2$ , hence;

For the enhanced bisection method 
$$\beta$$
 is computed as follows:

$$\beta = \left| \frac{\mathbf{d}_{e1}}{\mathbf{d}_{e2} - \mathbf{d}_{e1}} \right| \tag{21}$$

Hence;

$$d_{eop} = \frac{d_{e2} - d_{e1}}{\beta} = \frac{d_{e2} - d_{e1}}{\left|\frac{d_{e1}}{d_{e2} - d_{e1}}\right|}$$
(22)

B. The Fade Margin–Based Bisection Iteration Method For The Determination Of Optimal Path Length

Similarly, given two initial fade margin values,  $fm_e$  and  $fd_{me}$  and if  $fm_e < fd_{me}$ , the optimal effective

fade margin (fmop) can be computed using the bisection method as follows;

$$fmop = \frac{fd_{me} - fm_e}{\beta} \tag{23}$$

For the classic bisection method  $\beta = 2$ , hence;

$$fmop = \frac{fd_{me} - fm_e}{\beta} = \frac{fd_{me} - fm_e}{2}$$
(24)

The optimal effective path length  $(d_{eop})$  can be computed as follows;

$$d_{eop} = 10^{\left(\frac{(P_{T} + G_{T} + G_{R} - fmop - G_{d} - P_{S}) - 32.4 - 20\log(f*1000)}{20}\right)}$$
(25)

For the enhanced bisection method  $\beta$  is computed as follows:

$$\beta = \left| \frac{fm_e}{fd_{me} - fm_e} \right| \tag{26}$$

Hence;

$$fmop = \frac{fd_{me} - fm_e}{\beta} = \frac{fd_{me} - fm_e}{\left|\frac{fm_e}{fd_{me} - fm_e}\right|}$$
(27)

Numerical example, if  $fm_e = 19.96 \, \mathrm{dB}$ and  $fd_{me} = 124.83 \text{ dB}$  then

$$\beta = \left| \frac{fm_e}{fd_{me} - fm_e} \right| = \left| \frac{124.83}{124.83 - 19.96} \right| = \frac{124.83}{104.87}$$
$$= 1.190330885858682$$
$$fmop = \frac{fd_{me} - fm_e}{\left| \frac{fm_e}{fd_{me} - fm_e} \right|} =$$
$$\frac{124.83 - 19.96}{1.190330885858682} = \frac{104.87}{1.190330885858682} =$$
$$88.10155331250502$$

As can be seen, whereas bisection method will use  $\beta = 2$ the modified bisection method will use  $\beta = 1.190330885858682$ . The modified  $\beta$  value is expected to give rise to faster convergence of the algorithm.

$$\begin{cases} G_d = 0 & \text{for } V < -1 \\ G_d = 20 \log(0.5 - 0.62v) & \text{for } -1 \le V \le 0 \\ G_d = 20 \log(0.5 \exp(-0.95v)) & \text{for } 0 \le V \le 1 \\ G_d = 20 \log\left(0.4 - \sqrt{0.1184 - (0.38 - 0.1v)^2}\right) & \text{for } 1 \le V \le 2.4 \\ G_d = 20 \log\left(\frac{0.225}{v}\right) & \text{for } V > 2.4 \end{cases}$$

xii. Specified relative error tolerance,  $\epsilon_s$ ; Note:  $\epsilon_s = 0.01\% = \frac{0.01}{100} = 0.0001$ 

Step 2: Initialise parameters

**Step 2.1** Set k = 0; where k represent the iteration counter

Step 2.2  $fme_{(k)} = fm_s$ 

**Step 2.3**  $\in_s = 0.01\%$ 

Step 3 Compute the operating free space path loss, L<sub>FSP</sub> and set the initial operating free space path loss, L<sub>FSP(k)</sub>

Step 3.1 Compute  $P_R$ , the received signal power in dB as follows: $P_R = fme_{(k)} + P_S$ 

#### IV THE ENHANCED BISECTION METHOD FOR THE DETERMINATION OF THE OPTIMAL PATH LENGTH WHEN SINGLE KNIFE EDGE **DIFFRACTION LOSS IS INCLUDED**

In this paper, diffraction loss is specified in terms of LoS percentage clearance,  $P_c$  and the optimal path length is determined at the design time. As such, the diffraction loss does not change with the path length. Rather, after the determination of the optimal path length, the antenna mast height is selected such that the LoS clearance height from the LoS maintains the specified percentage clearance. In essence, what will be affected is the antenna mast height which will maintain the required LoS percentage clearance.

Step 1: Specify requisite link parameters values

Specify the following parameters

- i.  $P_s$  = the Receiver Sensitivity in dB
- ii.  $fm_s$  the specified (required) fade margin in dB
- iii.  $P_T$  = the Transmitter Power Output (dBm)
- iv.  $G_T$  = the Transmitter Antenna Gain (dBi)
- v.  $G_R$  = the Receiver Antenna Gain (dBi)
- vi.  $L_T$  = the Losses from Transmitter (cable, connectors etc.) (dB)
- vii.  $L_R$ = the Losses from Receiver (cable, connectors etc.) (dB)
- viii.  $L_M$  = the Miscellaneous Losses (fade margin, polarization misalignment etc.) (dB)
- ix. Specify the LOS percentage clearance, Pc

x. Compute diffraction parameter, V where; 
$$V = \left(\frac{(\sqrt{2})P_c}{100}\right)$$

xi. Compute single knife edge diffraction loss,  $G_d$  where

$$\begin{aligned} G_d &= 0 & \text{for } V < -1 \\ &= 20\log(0.5 - 0.62v) & \text{for } -1 \le V \le 0 \\ &= 20\log(0.5\exp(-0.95v)) & \text{for } 0 \le V \le 1 \\ &= 20\log\left(0.4 - \sqrt{0.1184 - (0.38 - 0.1v)^2}\right) & \text{for } 1 \le V \le 2.4 \\ &= 20\log\left(\frac{0.225}{v}\right) & \text{for } V > 2.4 \end{aligned}$$

Step 3.2 Compute LFSP, the Free Space Path Loss (dB) as follows:

LFSP = 
$$P_T$$
+  $G_T$ +  $G_R$ -  $P_R$ -  $G_d$  =  $P_T$  +  $G_T$ +  $G_R$  -  
 $fme_{(k)}$ -  $P_S$  -  $G_d$ 

Step 3.3 Set the initial operating free space path loss, L<sub>FSP(K)</sub>

$$L_{FSP(k)} = L_{FSP}$$

Step 3.4 Compute the operating fade depth,  $fd_m$  and set the initial effective operating fade margin,  $fd_{me(k)}$ 

Step 3.5 Compute d, the length of the link in km as follows:

$$d = 10^{\left(\frac{(\text{EFSP} - 32.4 - 20 \log(f \times 1000)}{20}\right)} = 10^{\left(\frac{(\text{P}_{\text{T}} + \text{G}_{\text{T}} + \text{G}_{\text{R}} - \text{G}_{d} - \text{P}_{\text{R}}) - 32.4 - 20 \log(f \times 1000)}{20}\right)}$$

**Step 3.6** Compute  $A_{Rain}$ , the operating rain fade depth in dB as follows:

$$\begin{aligned} \text{Step 3.6.1}\\ A_{R(h)} &= \left( \langle \gamma_{\text{Rpo}} \rangle_{\text{h}} \right) \text{d} = \left( \text{K}_{\text{h}} (\text{R}_{\text{po}})^{\alpha_{\text{h}}} \right) \text{d} \\ A_{R(\nu)} &= \left( \langle \gamma_{\text{Rpo}} \rangle_{\nu} \right) \text{d} = \left( \text{K}_{\nu} (\text{R}_{\text{po}})^{\alpha_{\nu}} \right) \text{d} \end{aligned}$$
$$A_{multipath} = 10(-0.00089h_{L}) - (10)\log \left( \frac{po}{\left\{ K(d^{3.1})\left(1 + |\varepsilon_{p}|\right)^{-1.29}(f^{0.8}) \right\}} \right)$$

**Step 3.8** Compute the operating fade depth,  $fd_m$  in dB as follows;

$$fd_m = \max(A_{multipath}, A_{Rain})$$
  
=  $max\left(A_{multipath}, \left(\max(A_{R(h)}, A_{R(v)})\right)\right)$ 

**Step 3.9** Set the initial effective operating fade margin,  $fd_{me(k)}$ , that is;  $fd_{me(k)} = fd_m$ 

**Step 4** Increase k by 1 that is; K = K+1

**Step 5** Use the values of  $fme_{(k-1)}$  and  $fd_{me(k-1)}$  to determine the adjustment value where the adjustment value is denoted as  $\Delta_{fme(k)}$ 

**Step 5.1** Set the two values  $x_{l(k)}$  and  $x_{u(k)}$  for fade margin where:

Step 5.1.1  

$$x_{l(k)} = minimum(fme_{(k-1)}, fd_{me(k-1)})$$

Step

 $x_{u(k)} = maximum(fme_{(k-1)}, fd_{me(k-1)})$ 

**Step 5.2** Compute the values of  $\beta$  as follows:  $\beta = \left| \frac{x_{l(k)}}{x_{u(k)} - x_{l(k)}} \right|$ 

- **Step 5.3** Determine the adjustment value for  $fm_{(k-1)}$ where the adjustment value,  $\Delta_{fm}$  is obtain as follows;  $\Delta_{fm} = \frac{x_{u(k)} - x_{l(k)}}{\beta} = \frac{x_{u(k)} - x_{l(k)}}{\left|\frac{x_{l(k)}}{x_{u(k)} - x_{l(k)}}\right|}$
- **Step 6** Determine the adjusted value for  $fd_{me(k)}$  where the adjusted value,  $fd_{me(k)}$  is given as;

**Step 6.1.1** 
$$fd_{me(k)} = fd_{me(k-1)} + \Delta_{fm}$$
, if  $x_{l(k)} = fd_{me(k-1)}$ 

**Step 6.1.2** 
$$fd_{me(k)} = fd_{me(k-1)} - \Delta_{fm}$$
, if  $x_{u(k)} = fd_{me(k-1)}$ 

**Step 7** Determine the adjusted value for optimal path length,  $d_{e(k)}$  where the adjusted value is given as:

**Step 7.1** If 
$$(A_{eR(h)} \ge A_{eR(v)})$$
 then  $(\gamma_{R_{po}} = \langle \gamma_{R_{po}} \rangle_{h})$   
otherwise  $(\gamma_{R_{po}} = \langle \gamma_{R_{po}} \rangle_{v})$ 

Step 7.2 
$$d_{e(k)} = \frac{\int d_{me(k)}}{\gamma_{R_{po}}}$$

Step 3.6.2 
$$A_{Rain} = \max\left(\left(K_{v}(R_{po})^{\alpha_{v}}\right) * \left(K_{h}(R_{po})^{\alpha_{h}}\right) * d\right) = \max\left(A_{R(h)}, A_{R(v)}\right)d$$

**Step 3.7** Compute the operating multipath fade depth,  $A_{multipath}$  in dB as follows:

**Step 8** Determine the adjusted value for effective free space path loss, 
$$L_{FSPe(k)}$$
 where the adjustment value is given as:  $L_{FSPe(k)} = 32.4 + 20 \log(f \times 1000) + 20 \log(d_{e(k)})$ 

**Step 9** Determine the adjusted value for effective fade margin where the adjusted value is given as;  $fm_{e(k)} = (LFSP + fm_s) - L_{FSPe(k)}$ 

**Step 10** Check if the optimal path length condition is met, that is if;

Step 10.1

$$\epsilon_{(k)} = \left| \frac{f d_{me(k)} - f m_{e(k)}}{f d_{me(k)}} \right| = \left| \frac{f d_{me(k)} - \left( (L_{\text{FSP}} + f m_s) - L_{\text{FSPe}(k)} \right)}{f d_{me(k)}} \right|$$

Step 10.2

5.1.2

If 
$$(\epsilon_{(k)} > |\epsilon_s|)$$
 Then

Else

Endif

Step 11 $fme_{op} = fme_{(k)}$ Step 11.2 $L_{FSPop} = L_{FSPe(k)}$ Step 11.3 $fd_{mop} = fd_{me(k)}$ Step 11.4 $d_{mop} = d_{e(k)}$ 

Step 12 Stop

#### **IV. RESULTS AND DISCUSSION**

#### A. Simulation of the Optimal Path Length Algorithms

In this paper, the classical bisection method and the enhanced bisection method for computing the optimal path length algorithms were considered. The parameters used for computing the optimal path length for a sample fixed point terrestrial LoS microwave link are presented in Table 1. The convergence cycle (n) is obtained for each of the two algorithms. Also, the simulation was conducted for microwave links at the following frequencies: 10 GHz, 20 GHz, 30GHz, 40 GHz, 50 GHz, 60 GHz, 70GHz, 80 GHz, 90 GHz, 100GHz, 150 GHz and 200 GHz.

#### Table 1.The parameters used for computing the optimal path length for a sample fixed point terrestrial LoS microwave link

S/N	Parameter Description	Parameter Value	Parameter Unit
1	Frequency (f)	10	GHz
2	Transmit power (P <sub>T</sub> )	10	dBm
3	Transmitter Antenna Gain (G <sub>T</sub> )	35	dBi
4	Receiver Antenna Gain (G <sub>R</sub> )	35	dBi
5	Fade Margin $(fm_s)$	20	dB
6	Receiver Sensitivity (P <sub>S</sub> )	-80	dBm
7	Rain Zone	N	
8	Point Refractivity Gradient (dN1)	-400	units
9	Link Percentage Outage (po)	0.01	%
10	Horizontal Polarization Rain Fade Constants: $k_h$	0.01006	
11	Horizontal Polarization Rain Fade Constants: $\alpha_h$	1.2747	
12	Vertical Polarization Rain Fade Constants: $k_v$	0.008853	
13	Vertical Polarization Rain Fade Constants: $\alpha_v$	1.263	
14			

15	Rain Rate $(R_{po})$	95	mm/h
16	Transmitter antenna height $(h_t)$	295	m
17	Receiver antenna height $(h_r)$	320	m

## B. Results for The Enhanced Bisection Method

In Table 2 to Table 4, as well as Figure 2 to Figure 4, the frequency is 10 GHz and the rain zone is N, with percentage availability of 99.99%. The convergence cycle is 5. That means, as shown in Table 2, Table 3, and Table 4, (as well as, Figure 2, Figure 3, and Figure 4), the Enhanced Bisection algorithm is iterated for 5 times before the optimal path length is obtained. Also, the optimal path length is 8.636 km, the optimal free space path loss is 131.17 dB, the optimal fade margin the system can accommodate is 28.83 dB and the optimal fade depth value is 28.83 dB. In essence, for the Enhanced Bisection algorithm, at the optimal path length, a maximum fade depth of 28.83 dB can be accommodated by the link which is the same with the optimal fade depth value of 28.83 dB. It can be recalled from Table 4.8 and Figure 4.8 that the initial fade margin specified for the system is 19.60 dB. At this initial point, in Table 4.8 and Figure 4.8, the initial maximum path length is 23.9883 km, the initial path loss is 140.40 dB, the initial fade depth is 140.04 dB while the received signal power is -60.04 dB. At the optimal point, the path maximum path loss has reduced by 8.87 dB to a value of 131.17 dB while the received signal power has increased the same value of 8.87 dB to a value of -51.17 dB. From Table 2 and Figure 2, it will be noticed that the rain fading is equal to the effective fade depth. In essence, for the given frequency, rain zone and percentage availability, the rain fading is greater than the multipath fading and hence, determines the effective fade depth that will be experienced in the link.

### Table 2: Enhanced Bisection method: Rain fading, multipath fading, free space path

loss, effective fade margin, effective maximum depth and effective path length vs number of iterations (n)

Number Of Iterations (n)	Effective Rain Fading (dB)	Multipath Fading (dB)	Free Space Path Loss (dB)	Effective Fade Margin (dB)	Effective Fade Depth (dB)	Effective Path Length (km)
0	80.09	28.95	140.04	19.96	80.09	23.98833
1	34.95	14.95	132.84	27.16	34.95	10.46630
2	29.10	11.75	131.25	28.75	29.10	8.71648
3	28.84	11.59	131.17	28.83	28.84	8.63773
4	28.83	11.58	131.17	28.83	28.83	8.63583
5	28.83	11.58	131.17	28.83	28.83	8.63579
6	28.83	11.58	131.17	28.83	28.83	8.63579
7	28.83	11.58	131.17	28.83	28.83	8.63579
8	28.83	11.58	131.17	28.83	28.83	8.63579
9	28.83	11.58	131.17	28.83	28.83	8.63579



Figure 2: Enhanced Bisection method: Rain fading, multipath fading, free space path loss, effective fade margin, effective maximum depth and effective path length vs number of iterations (n)

 Table 3: Enhanced Bisection method: Initial and optimal values for free space path

 loss, fade depth, fade margin, received power, path length and convergence

cycle

			cycic			
	n	Free Space Path Loss (in dB)	Fade Depth (in dB)	Fade Margin (in dB)	Received Power (in dBm)	Path Length (in km)
Initial Value	0	140.04	80.09	19.96	-60.04	23.9883
<b>Optimal Value</b>	5	131.17	28.83	28.83	-51.17	8.6358



Figure 3: Enhanced Bisection method: Initial and optimal values for free space pathloss, fade depth, fade margin, received power, path length and convergence cycle

mber Of Iterations		
( <b>n</b> )	Differential Fade Depth	Effective Path Length (de)
0	60.1344	23.9883
1	7.7817	10.4663
2	0.3502	8.7165
3	0.0084	8.6377
4	0.0002	8.6358
5	0.0000	8.6358
6	0.0000	8.6358
7	0.0000	8.6358
8	0.0000	8.6358
9	0.0000	8.6358
10	0.0000	8.6358

Table 4: Enhanced Bisection method: Differential fade depth and effective path

length vs number of iterations (n)



Figure 4: Enhanced Bisection method: Differential fade depth and effective path length vs number of iterations (n)

# *C. Effect of Frequency on the convergence cycle Enhanced Bisection method*

Table 5 to Table 7 (as well as, Figure 5 to Figure 7) show how the various link parameters vary with frequency which is varied from 10 GHz to 200 GHz. Specifically, in Table 5 and Figure 5 show that the convergence cycle for the Enhanced Bisection algorithm varied from 5 to 8 cycles as the frequency is varied from 10 GHz to 200GHz. Essentially, the convergence cycle increases with increasing frequency. On the other hand, the optimal path length decreases from 8.64 km at 10 GHz to 0.06 km at 200 GHz. The optimal fade depth and optimal path loss decreased from 28.83 dB and 131.17 dB at 10 GHz to 45.77 dB and 114.23 dB at 200 GHz respectively. Also, in Table 7 and Figure 7, the rain fading is the dominant fading for all the frequencies considered, namely, 10 GHz and to 200 GHz.

The results from Table 8 and Figure 8 show that classical Bisection method has a convergence cycle of 17 to 13 as the frequency is varied from 10 GHz to 200GHz. On the other hand, the convergence cycle for the Enhanced Bisection algorithm varies from 5 to 8 cycles for all frequencies. In essence, the Enhanced Bisection method developed in this paper has proven to be the better algorithm for determination of the optimal path length of LOS terrestrial microwave link.

	f (GHz)	Convergence Cycle	Initial Path Length (km)	Optimal Path Length (km)
-	10	5	23.99	8.64
	20	6	11.99	2.88
	30	6	8.00	1.72
	40	6	6.00	1.31
	50	6	4.80	1.14
	60	5	4.00	1.04
	70	5	3.43	0.98

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I anie 5.	Ennancea	Risection method.	Inifiali	narn lengrn	ontimal	natn lengtn	i ana con	vergence c	vcie vs trea	nencv
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-	80	4	3.00	0.94
	90	4	2.67	0.90
	100	5	2.40	0.87
	150	8	1.60	0.09
	200	8	1.20	0.06



Figure 5: Enhanced Bisection method: Initial path length, optimal path length and convergence cycle vs frequency Table 6: Enhanced Bisection method: Optimal path length, optimal fade depth, optimal path loss and convergence cycle vs frequency

		Optimal Path Length	Optimal Fade Depth	Optimal Path loss
f(GHz)	<b>Convergence Cycle</b>	( <b>km</b> )	( <b>dB</b> )	( <b>dB</b> )
10	5	8.64	28.83	131.17
20	6	2.88	32.34	127.66
30	6	1.72	33.32	126.68
40	6	1.31	33.16	126.84
50	6	1.14	32.46	127.54
60	5	1.04	31.64	128.36
70	5	0.98	30.82	129.18
80	4	0.94	30.07	129.93
90	4	0.90	29.37	130.63
100	5	0.87	28.74	131.26
150	8	0.09	44.51	115.49
200	8	0.06	45.77	114.23



Figure 6: Enhanced Bisection method: Optimal path length, optimal fade depth, optimal path loss and convergence cycle vs frequency

Table 7:	Enhanced	Bisection method: Optimal rain fading, optimal multipath fading and optimal effective fading va
		frequency

f (GHz)	Optimal Rain Fading	Optimal Multipath Fading	Optimal Effective Fading
	( <b>dB</b> )	( <b>dB</b> )	( <b>dB</b> )
10	28.83	11.58	28.83
20	32.34	0.00	32.34
30	33.32	0.00	33.32
40	33.16	0.00	33.16
50	32.46	0.00	32.46
60	31.64	0.00	31.64
70	30.82	0.00	30.82
80	30.07	0.00	30.07
90	29.37	0.00	29.37
100	28.74	0.00	28.74
150	44.51	0.00	44.51
200	45.77	0.00	45.77



Figure 7: Enhanced Bisection method: Optimal rain fading, optimal multipath fading and optimal effective fading vs frequency

f (GHz)	Convergence Cycle for Classical Bisection method	Convergence Cycle for Enhanced Bisection method
10	17	5
20	16	6
30	16	6
40	15	6
50	15	6
60	15	5
70	14	5
80	14	4
90	14	4
100	14	5
150	13	8
200	13	8



Figure 8: Comparison of the convergence cycle of the seven methods

# **IV. CONCLUSION**

In this paper the classical Bisection Method and enhanced Bisection method were used to determine the optimal path length for fixed point terrestrial microwave communication link with knife edge diffraction loss. Sample numerical microwave link was used to compare the convergence cycle of the two algorithms and the impact of frequency on the performance of the algorithms. In all, the enhanced Bisection method performed better than the classical Bisection method.

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