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Estimating Option Price Sensitivities and Determining Hedging Strategies

[1]Gifty Osei, [2]Nora Irene Ghartey, [3]Charlotte Osei Poku, [4]Y.E. Ayekple, [6]Vincent K. Dedu
Department of Statistics and Actuarial Science, Kwame Nkrumah University of Science and Technology, Kumasi, Ghana
gifty18osei@gmail.com; noraireneghartey@gmail.com; charllynhyira@gmail.com; yeayekple.cos@knust.edu.gh; vkdedu.cos@knust.edu.gh.

Abstract - The derivatives market is very important in the world's financial market. Most option traders engage in options trading without adequate understanding of the price sensitivities with market factors. The leads to engagement in highly risky investments for investors in some developing countries. This paper focuses on the model used to compute the theoretical value of a European option and derive the various price sensitivities to some variables and parameters known as the Greeks. We delve into the options price sensitivities as well as some beneficial strategies that can be implored to reduce risk during investment. The Greeks were computed using the black-Scholes model in Automatic differentiation in Julia.

Keywords - Stochastic Processes, Black-Scholes Model, Delta Hedging, Delta-Gamma Hedging, Long Position, Short Position, Risk Analysis, The Investor, European Options.

I. INTRODUCTION

The financial market of the world has gone through many changes qualitatively in the last four decades due to phenomenal advancement of derivatives. An increasingly large number of organizations now consider derivatives to play a key role in implementing their financial strategies. Various types of derivatives being traded today are futures, options, interest rate swaps and mortgage derivatives. Options first emerged in the 1600s in Holland because of Tulips, which were extremely popular as a status symbol for the Dutch Aristocrat. In 1791, the New York Stock Exchange opened and after its crash in 1929, there was the creation of a regulation body, the Securities and exchange commission to regulate over-the - counter markets.

Options were also used in ancient Greece for speculation on the olive harvest; however, recent option contracts are commonly referring to as equities. In simple terms, a stock option contract gives the holder the right to buy or sell a specified number of shares for a predetermined price over a defined period. Most options are executed electronically and go through a clearing agency called the options clearing corporation (OCC). Trading in options without an understanding of the Greeks can be detrimental to the trader. Unfortunately, many options traders are engaging in trades without a fundamental understanding of the Greeks: Delta, Gamma, Theta, Vega, Rho and Lambda or the concept underlying them. By buying two options of the same type and strike price with different expiration at the same time, open-minded options traders with a position can profit from a sideways market. In this paper, we will look at the five essential Greeks in Options pricing.

We will outline how Theta: which measures the effect of time decay's impacts on the position, but the hidden cost of this strategy actually stems from Vega: which measures the effect of the implied volatility. Delta: a measure of the impact of the underlying asset, Gamma: measures the rate of change of delta with the price of the underlying asset, Rho: a measure of the impact of changes in interest rates on an option's price and Lambda: the impact of the continuous dividend yield on the underlying security. In recent times, we have seen several new players enter in the financial Marketplace because of the use of computerized trading systems and easy access to internet, which has phenomenally made it easy for an investor to place an option trade [15].

Derivatives are financial instruments whose value depends on some basic underlying cash products, such as interest rate, equity indices, commodities, foreign exchange, or bonds [1]. An option is an example of a derivative. Black and Scholes (1973) derived the standard equations for pricing European call and put options and there on there have been many models developed for pricing financial options. Many have also worked extensively to obtain solution to the Black-Scholes model.

In Reference [4], they tested the random walk hypothesis for weekly stock market returns by comparing variance estimators derived from data sampled at different frequencies. In contrast, the random walk model is strongly rejected for the entire sample period (1962-1985) and for all sub-periods for a variety of aggregate returns indexes and size-sorted portfolios. Although the rejections were largely due to the behavior of small stocks, they cannot be ascribed to either the effects of infrequent trading or time-varying volatilities. Moreover, the rejection of the random walk cannot be interpreted as supporting a mean-reverting stationary model of asset prices, but is more consistent with a specific non-stationary alternative hypothesis. For comparison, however, they estimated models using the Ordinary least square, Radial basis function networks, multi-layer and the projection pursuit. At the end they concluded that although parametric derivative was of finding pricing formula is preferred, the non-parametric learning network approach can be useful when the parametric fails.

Reference [5] revealed in his paper the two main reason for financial derivative that is; Hedging and Speculating. Hedging means minimizing or eliminating the risk of the price movement of an asset. Investors who are using financial derivatives for hedging purposes are looking to reduce exposure to the underlying asset. Speculation, on the other hand, is when an investor wants to increase exposure to an asset. There are many reasons that an investor would choose to gain exposure to an asset by purchasing a financial derivative rather than the underlying asset.

In-revisiting the Greeks for European and American options, Gobet et al addressed issues of defining options as a general stochastic differential equation and represented the Greeks as expectations in order to allow easy computations using the Monte Carlo Simulations. Gobet used the Markovian structure to derive simple formulas generally. At the end of their paper, they had developed simple arguments to get simple representations [2].

According to [3], financial derivatives are innovative instruments in the market and its judicial use in right proportions enables a corporate manager to minimize risk and optimize returns. Denise et al studied the efficacy of the commonly used option Greeks (Delta, Gamma, Vega, rho, kappa) and their significance in managing various types of risk associated with an options contract. The individual potentials of these five option Greeks for an European option using the Black-Scholes model was studied. To strengthen the derivative market, the players must have an ideal knowledge in understanding the Greeks and their prudent applications for hedging against adverse price movements. The most common reason is that, financial derivatives can increase leverage allowing a small move in the price of the underlying asset to provide a much larger movement in the price of the derivative

Reference [10] Implemented dynamic delta hedging strategies based on several option-pricing models. The researcher analyzed different subordinated option pricing models and examined delta-hedging costs using ex-post daily prices of S&P 500 as well as comparing the performance of each subordinated model with the Black-Scholes Model.

The preferred numerical method to derive the sensitivities of a computer program is Algorithmic Differentiation (AD) [7]. This technique returns mathematically exact derivatives with machine accuracy up to an arbitrary order by exploiting elemental symbolic differentiation rules and the chain rule. AD distinguishes between two basic modes: the forward mode and the reverse mode.

This paper dealt with the computation of second or higher order Greeks of financial securities. It combined two methods, Vibrato and automatic differentiation and compares with other methods. They showed that this combined technique is faster than standard finite difference, more stable than automatic differentiation of second order derivatives and more general than Malliavian Calculus. The paper presents a generic framework to compute any Greeks and present several applications on different types of financial contracts: European and American options, multidimensional Basket Call and stochastic volatility models such as Hesston's model. We give also an algorithm to compute derivatives for the Long staff-Schwartz Monte Carlo method for American options. We also extend automatic differentiation for second order derivatives of options with non-twice differentiable payoff [8].

II. METHODOLOGY

We focus on the models used to compute the theoretical value of an European option and derive the various price sensitivities to some variables and parameters known as the Greeks. Hedging strategies for a portfolio of stocks is determined. The main model considered here is the Black-Scholes-Merton model, popularly known as the Black-Scholes formula which is the first widely used mathematical equation for pricing an European option. We will finally look at the basic concept of Dual numbers in Julia and its properties which will be used in the computation of the Greeks.

A. Black-Scholes PDE

The Black-Scholes model is a continuous-time formulation for deriving the formulae for European option

$$rf(S_t) = \Theta + \Delta(r-q)S_t + \frac{\sigma^2}{2}\Gamma S_t^2; \qquad (1)$$

B. Geometric Brownian Motion

Geometric Brownian motion $ S(t)$ is a non-negative Brownian motion unlike the main Brownian Motion defined by;

$$S(t) = S_0 e^{X(T)} \qquad (2)$$

where $X(t) = \sigma B(t) + \mu t$ is Brownian motion with drift and $S(0) = S_0 > 0$ is the initial value.
Taking the log of the equation above (eqn. 2) yields the Brownian Motion equation. Hence $S(t)$ has a log normal distribution.

C. Greeks

The Greeks which are also known as risk sensitivities or hedge parameters are values that show the measure of responsiveness of an options price to changes in some underlying variables or parameters. Most of these measures are represented by Greek letters such as delta, gamma, theta, rho, etc. hence their name (the Greeks).

- Delta

Delta, which is the first derivative of the option price with respect to the underlying asset measures the rate at which the theoretical price of an option will change given changes in the price of the asset and it is given by;

$$\Delta = \frac{\partial V}{\partial S}$$

The delta of the underlying asset price Δ_s is equal to 1. This is simply because the derivative of any variable with respect to itself is 1 and it means that;
for example,

$$\Delta_s = \frac{\partial s}{\partial s} = 1$$

This measure takes on values between $-1 \leq \Delta \leq 1$ and the delta of a long call is always in the range $0 \leq \Delta_c \leq 1$ whereas that of the long put within the range, $-1 \leq \Delta_p \leq 0$

The Delta for put and call options in the Black-Scholes model, for non-dividend and dividend paying stocks is given as;

$$\Delta_c = N'(d_1)$$
$$\Delta_p = -N'(-d_1)$$

and

$$\Delta_c = e^{-D(T-t)}N'(d_1)$$
$$\Delta_p = e^{-D(T-t)}N'(d_1 - 1)$$

respectively.

- Gamma

The Gamma of an option is the second derivative of its value with respect to the price of the underlying asset. This implies that gamma is also a measure of the rate of change of the delta of an option with respect to changes in the price of the underlying.

$$\Gamma = \frac{\partial^2 V}{\partial S^2} = \frac{\partial V}{\partial S}$$

Since Gamma is calculated based on Delta and its range is $-1 \leq \Gamma \leq 1$ gamma can not be greater than 1 and hence is within the same range as Delta. It also tells us how often a position should be re-balanced in order to maintain a delta-neutral position.

Gamma is very important in hedging risk as it gives traders a wider perspective to the behaviour of an option when considering that of the price of the underlying asset. A higher gamma value requires the position to be readjusted often which will depend on the liquidity of the market however the cost of transaction or the round-trip cost may increase which makes continuous re-hedging expensive and unrealistic. The calculation of gamma is complex and would require computer programs to compute its precise value. Below are formulas derived for the gamma of a call (Γ_c) and put (Γ_p) of an European option using the Black-Scholes model.

$$\Gamma_c = \frac{e^{-D(T-t)}N'(d_1)}{\sigma S\sqrt{T-t}}$$

$$\Gamma_p = \frac{e^{-D(T-t)}N'(d_1)}{\sigma S\sqrt{T-t}}$$

Gamma is highest when the option is at-the-money and lowest when it is either out-of-the-money or in-the-money.

- Vega

This sensitivity measures the rate of change of the theoretical value of an option to changes in implied volatility of the underlying asset. Implied volatility is a parameter in the Black-Scholes equation which has to be estimated because the future is uncertain. Investors sometimes use historical volatility to predict the direction implied volatility will move in but it is mainly the market that determines this. When uncertainty is high, implied volatility turns to be higher and lower when in a steady market as the tendency of price fluctuation is low. So long term options tend to have a higher Vega than short-term ones. Vega is an indicator of the amount by which an option price will change given a one percent change in implied volatility. The Vega of an option is given by;

$$v = \frac{\partial V}{\partial \sigma}$$

The Vega for a call and put option are the same and the formula is;

$$v = S\sqrt{T-t}e^{-D(T-t)}N'(d_1)$$

- Theta

Theta is the sensitivity of an option price to time. It is always negative because the time to maturity decreases over the period of holding the option. It is a first derivative Greek given by;

$$\theta = \frac{\partial V}{\partial \tau}$$

Theta for call and put options are given as;

$$\theta_C = -\frac{\sigma S e^{-D(T-t)}N'(d_1)}{2\sqrt{T-t}} + DSN'(d_1)e^{-D(T-t)} - \tau K e^{-D(T-t)}N'(d_2)$$

$$\theta_p = -\frac{\sigma S e^{-D(T-t)}N'(-d_1)}{2\sqrt{T-t}} - DSN'(-d_1)e^{-D(T-t)} + \tau K e^{-D(T-t)}N'(-d_2)$$

- Rho

Rho is the option's sensitivity to interest rates. It is the first derivative of an option with respect to the risk-free interest rates and is denoted by ρ.

$$\rho = \frac{\partial V}{\partial r}$$

Similar to Vega, the effect of interest rate fluctuations is much greater in longer dated options than shorter ones and more evident in situations where an option is currently at-the-money with longer time to expiration. Long calls have a positive rho because higher interest rates increase call premiums while higher interest rates tend to decrease the value of long put. However, the impact of changes in interest rates has a minimal effect on the pricing of an option when using the Black-Scholes model which makes rho the least significant among the commonly used Greeks. The formula of rho for call and put options are given below.

$$\rho_c = K(T-t)e^{-r(T-t)}N(d_2)$$

$$\rho_p = -K(T-t)e^{-r(T-t)}N(-d_2)$$

$$\frac{\partial^3 V}{\partial S^3} = -\frac{e^{-D(T-t)}N'(d_1)}{\sigma^2 S^2 (T-t)}(d_1 + \sigma\sqrt{(T-t)}$$

III. HEDGING STRATEGIES

In Financial markets, an investor or market maker with the intention of making profit wants to understand certain conditions with respect to hedging with the options Greeks. To hedge with the Greeks - (Delta, Gamma, Vega), a market maker needs to explicitly specify whether the Greeks applies to a risky asset like that of the stock prices or a parameter in the model used.

Taylor Expansion for Options: Hedging begins with Taylor series as;

$$\Delta C_t = \frac{\partial C_t}{\partial t} + \frac{\partial C_t}{\partial S_t}\Delta S_t + \frac{1}{2}\frac{\delta^2 C_t}{\delta S_t^2}\left(\Delta S_t{}^2\right) + \frac{\delta C_t}{\delta \Sigma}\Delta\sigma + error$$

Where $\Delta C_t = C_{t+\Delta t} - C_t$ and $\Delta S_t = S_{t+\Delta t} - S_t$

A. Delta Hedging

From the above Taylor expansion, we exclude all the terms except the first two terms i.e;

$$\Delta C_t = \frac{\delta C_t}{\delta t}\Delta t + \frac{\delta C_t}{\delta S_t}\Delta S_t + error$$

To hedge in a long position of a call option we need to sell $n = \frac{\delta C_t}{\delta t}$ units of stock.

$$C_t - \frac{\delta C_t}{\delta t}$$

if the portfolio changes over time Δt we get;

$$C_t - \frac{\delta C_t}{\delta t} = \frac{\delta C_t}{\delta t}\Delta t + error.$$

Since the right-hand side is deterministic or small, we have a risk-less portfolio (delta neutral). This process has a background of the arbitrage free principle in the Black-Scholes model. but in reality, one cannot delta -hedge continuously hence the introduction of the Gamma Hedging.

A portfolio is Delta-neutral if $\Delta(portfolio) = 0$

$$V_{port} = S_n(s) + Cn(c; s)$$

$$\Delta(portfolio) = n(s) + \Delta(c)n(c; s)$$

B. Gamma Hedging

Under this type of hedging, we use the first 3 terms in the Taylor expansion to get;

$$\Delta C_t = \frac{\partial C_t}{\partial t} + \frac{\partial C_t}{\partial S_t}\Delta S_t + \frac{1}{2}\frac{\delta^2 C_t}{\delta S_t^2}\left(\Delta S_t{}^2\right)$$

Over time interval $[t, \Delta t]$, delta will change so we hedge change in both delta and the price. We include a distinct call option with a different strike price and maturity date. Choosing n shares of the stock and m shares of C_{t^*};

$$C_t - nS_t - mC_{t^*}\Delta C_t - n\Delta S_t - mC_{t^*} = \left[\frac{\delta C_t}{\delta t} - \frac{m\delta C_{t^*}}{\delta t}\right]\Delta t + \left[\frac{\delta C_t}{\delta S_t} - n - \frac{m\delta C_{t^*}}{\delta S_t}\right]\Delta S_t + \frac{1}{2}\left[\frac{\delta^2 C_t}{\delta S_t^2} - \frac{m\delta^2 C_{t^*}}{\delta S_t^2}\right](\Delta S_t)^2 + error$$

Choosing n and m such that $\left(\frac{\delta C_t}{\delta S_t} - n - m\frac{\delta C_{t^*}}{\delta S_t}\right) = 0$ and $\left(\frac{\delta^2 C_t}{\delta S_t^2} - m\frac{\delta^2 C_t}{\delta S_t^2} - m\frac{\delta^2 C_{t^*}}{\delta S_t^2}\right) = 0$ then we have;

$$m = \frac{\frac{\delta^2 C_t}{\delta S_t^2}}{\frac{\delta^2 C_{t^*}}{\delta S_t^2}}$$

$$n = \frac{\delta C_t}{\delta S_t} - \frac{\frac{\delta^2 C_t}{\delta S_t^2}}{\frac{\delta^2 C_{t^*}}{\delta S_t^2}} * \frac{\delta C_{t^*}}{\delta S_t}$$

This is a delta and gamma neutral position. Then find the strategy under delta-gamma hedge,

- Use another option on same stock to neutralize Γ of existing position.

- Calculate Δ of the new portfolio.

$$\Delta_p = n_s\Delta_s + n_1\Delta_1 + n_2\Delta_2 = 0$$

$$\Gamma_p = n_s\Gamma_s + n_1\Gamma_1 + n_2\Gamma_2 = 0$$

C. Vega Hedging

Black-Scholes Model assume a constant volatility but in reality, this is not true. Vega is a procedure for corrections. Vega Hedging does not work because the original call option model does not include volatility risk.

Calculating the Value of a Portfolio

To find the profit that the investor or market maker makes we used the following;

- The value of the portfolio for short position give as;

$$S_0 e^{\tau T} + Payoff$$

- The cost of cresting the portfolio too is given as;

$$S_t + Ce^{\tau T}$$

IV. RESULTS AND ANALYSIS

We present the stock price data of some selected companies from the Chicago Board Options Exchange (CBOE); one of the world's options market, focusing on interest rates, indexes and equities. CBOE is in charge of determining the future of financial investment to create more value for market investors. This data contains information for 5 years spanning from 2014 to 2018. We compute option Greeks and analyze the results from the investor's perspective and that of the market maker using methods described in the preceding analysis. Estimation of the Greeks and option premium was done using the Black-Scholes package and forward diff in Differential Toolbox in Julia. All the analysis for the hedging strategies was done using Microsoft Excel.

A. Data Description

In this study, we selected stocks from 9 different companies on the CBOE and the excerpt of the data is shown below in Table I

The volatility is computed with $stddev[\ln(\frac{P\_\{t-1\}}{P\_\{t\}})^{-1}]$ and two different strikes are selected for options traded on the underlying stock respectively. These were used to compute the option price sensitivities and premium together with a risk-free rate of 5% per annum over a 6 months period.

B. ANALYSIS

Below are the results obtained for the various Greeks of an European call and put option computed using the Julia programming language. Tables II and III show results for option I whereas tables IV and V show results obtained for option II.

TABLE I

STOCKS	S0	SEND	STRIKE I	VOLATILITY
AFL	43.83	42.89	44	0.006781429
APA	44.3	44.68	45	0.017288656
COG	28.9	23.52	30	0.015676631
HBI	21.3	22.05	20	0.013268003
MET	50.18	43.58	52	0.011002905
OXY	73.98	82.52	73	0.010574853
PRU	115.45	110.18	115	0.010834677
PFG	70.38	52.86	70	0.01028589
WDC	81.38	78.04	79	0.016047951

Fig 1: The Original Data From CBOE

TABLE II

STOCKS	PREMIUM	DELTA	GAMMA	RHO	THETA	VEGA
AFL	0.3732	0.9616	0.3962	20.8878	-1.0492	2.5811
APA	0.1505	0.3950	0.7110	8.6741	-0.6369	12.0614
COG	0.0013	0.0122	0.0990	0.1757	-0.0188	0.6479
HBI	1.5454	1.0000	0.0000	9.8773	-0.4878	0.0000
MET	0.0002	0.0014	0.0118	0.0351	-0.0035	0.1629
OXY	1.8759	0.9997	0.0020	36.0412	-1.7807	0.0565
PRU	1.8666	0.9832	0.0471	55.8230	-2.7940	3.4039
PFG	1.2402	0.9928	0.0392	34.3155	-1.7051	0.9978
WDC	3.3495	0.9999	0.0004	39.0110	-1.9271	0.0236

Fig 2: European Call Option 1

TABLE III

STOCKS	PREMIUM	DELTA	GAMMA	RHO	THETA	VEGA
AFL	0.0032	-0.0384	0.3962	0.8422	0.0241	2.5811
APA	0.2983	-0.6050	0.7110	-13.5498	0.4607	12.0614
COG	0.7332	-0.9878	0.0990	-14.6402	0.7129	0.6479
HBI	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MET	1.1820	-0.9986	0.0118	-25.6458	1.2649	0.1629
OXY	0.0000	-0.0003	0.0020	-0.0108	-0.0001	0.0565
PRU	0.0053	-0.0168	0.0471	-0.9714	0.0111	3.4039
PFG	0.0012	-0.0072	0.0392	-0.2550	0.0023	0.9978
WDC	0.0000	-0.0001	0.0004	-0.0042	-0.0002	0.0236

Fig 3: European Put Option 1

TABLE IV:

STOCKS	NEW PREMIUM	DELTA	GAMMA	THETA	VEGA	RHO
AFL	0.000107	0.00177	0.02700	-0.00311	0.17587	0.03874
APA	0.853457	0.942022	0.21412	-1.07229	3.63248	20.43906
COG	0.294061	0.790403	0.89861	-0.64906	5.88285	11.2743
HBI	0.001200	0.016761	0.20843	-0.01711	0.62734	0.177903
MET	0.077815	0.311205	0.90517	-0.52169	12.53916	7.769215
OXY	0.900431	0.947262	0.19451	-1.76788	5.62888	34.58902
PFG	0.353656	0.689845	0.68935	-1.37088	17.56100	24.09883
PRU	0.298862	0.450166	0.44752	-1.62617	32.31342	25.8364
WDC	2.363082	0.995352	0.01467	-1.95450	0.77965	39.31933

Fig 4: New Option with different Strike price

TABLE V

STOCKS	NEW PREMIUM	DELTA	GAMMA	THETA	VEGA	RHO
AFL	0.617885	-0.99823	0.026999	1.094535	0.175866	-22.1851
APA	0.013506	-0.05798	0.214124	0.00096	3.632482	-1.29096
COG	0.038184	-0.2096	0.898606	0.058306	5.882852	-3.04776
HBI	0.431224	-0.98324	0.208433	0.519514	0.627336	-10.6871
MET	0.271962	-0.6888	0.905169	0.722304	12.53916	-17.4179
OXY	0.012331	-0.05274	0.194513	0.037128	5.628884	-1.95693
PFG	0.102371	-0.31015	0.689348	0.360959	17.561	-10.9655
PRU	0.413083	-0.54983	0.447517	1.227698	32.31342	-31.9457
WDC	0.001352	-0.00465	0.014671	-0.00314	0.77965	-0.1898

Fig 5: New European Put Option

C. HEDGING STRATEGIES FOR THE INVESTOR

We study an investor who takes a long or short position in a portfolio of 5 stocks and we compare the value of his portfolio for a six-month period in a hedged and an un-hedged situation.

- Long Position

An investor who takes a long position in a portfolio of stocks is exposed to the risk of price reduction. The value of his portfolio at the end of the period is uncertain and as such, hedging his position will be profitable to him. If the price of his underlying asset increases at the end of the period, he gains but he cannot control losses when the price of the asset takes a downward turn. It is safe but that of a hedged portfolio will provide some form of security for the investor. We can see from table VI that; the unhedged portfolio makes a loss of $1781.00 at the end of the period. Delta hedging portfolio 1a, using the formulas described in the methodology, we obtained a portfolio profit of $30745.05 which is a vast improvement from that of the unhedged portfolio. In table VIII we delta-gamma hedge and the portfolio profit of $11805.05 made is much lesser than that of portfolio 1b. This can be attributed to reduction in the portfolio value of AFLAC Inc and Prudential stocks which is the price paid for eliminating both the delta and gamma risk associated with this portfolio. See table XII and XIII for the risk analysis of the various hedged stages.

TABLE VI

UNHEDGED PORTFOLIO LONG					
STOCKS	**AFL**	**APA**	**COG**	**MET**	**PRU**
NUMBER OF STOCKS	**100**	**100**	**100**	**100**	**100**
S_0	43.83	44.3	28.9	50.18	115.45
S_END	42.89	44.68	23.52	43.58	110.18
GAIN	-94	38	-538	-660	-527
PORTFOLIO PROFIT	**-1781**				

Fig 6: Un-hedged Portfolio for the Investor

TABLE VII

PUT OPTIONS					
STOCKS	**AFL**	**APA**	**COG**	**MET**	**PRU**
NUMBER OF STOCKS	**100**	**100**	**100**	**100**	**100**
STRIKE 1	44	45	30	52	115
DELTA	-0.0384	-0.6050	-0.9878	-0.9986	-0.0168
PAYOFF (LONG PUT)	1.11	0.32	6.48	8.42	4.82
NUMBER OF PUTS	2607	165	101	100	5959
VALUE OF PORTFOLIO	7182.923	4520.893	3008.009	5201.183	39739.345
COST OF PORTFOLIO	4446.683	4535.646	3001.512	5200.977	11722.484
PROFIT	**2736.239**	**-14.753**	**6.497**	**0.206**	**28016.861**
PORTFOLIO PROFIT	**30745.052**				

Fig 7: Delta Hedging

TABLE VIII

STOCKS	AFL	APA	COG	MET	PRU
NUMBER OF STOCKS	**100**	**100**			
DELTA I	-0.03835624	-0.604998	-0.9877911	-0.998597	-0.0167782
DELTA II	-1.00	-0.06	-0.21	-0.69	-0.55
GAMMA I	0.40	0.71	0.10	0.01	0.05
GAMMA II	0.03	0.21	0.90	0.91	0.45
NO. OF PUT I	-6.84	242.43	103.66	101.05	-2430.82
NO. OF PUT II	100.44	-809.98	-11.42	-1.31	256.07
VALUE OF PORTFOLIO	4562.45	4590.77	3115.41	5273.00	23326.03
COST OF PORTFOLIO	4500.97	4558.95	3003.31	5202.06	11797.33
PROFIT	**61.48**	**31.83**	**112.10**	**70.94**	**11528.70**
PORTFOLIO PROFIT	**11805.05**				

Fig 8: Delta-Gamma Hedging

- Short Position

Again, an investor who takes a short position in a portfolio of 5 stocks as shown in table IX below, will realize a total profit of $1781.00 at the end of the investment period. Though there is a gain, the portfolio is exposed to risk of upward price movement of the stock. The investor makes a profit of $1844.04 when he holds a delta neutral position and a portfolio profit of $2500.81 when he further hedges against delta risk. To delta hedge, we find the number of call I to be purchased which will make our portfolio delta 0 for each stock from the 5 selected companies. The investor as shown in the tables below, is better-of when he holds a hedged portfolio and table XIII shows the risk analysis of the portfolio for the various hedged and unhedged positions.

TABLE IX

UNHEDGED PORTFOLIO SHORT					
STOCKS	**AFL**	**APA**	**COG**	**MET**	**PRU**
NUMBER OF STOCKS	**100**	**100**	**100**	**100**	**100**
S_0	43.83	44.3	28.9	50.18	115.45
S_END	42.89	44.68	23.52	43.58	110.18
GAIN	94	-38	538	660	527
PORTFOLIO PROFIT	**1781**				

Fig 9: Unhedged Portfolio Short Position

TABLE X

STOCKS	AFL	APA	COG	MET	PRU
NUMBER OF STOCKS	**100**	**100**	**100**	**100**	**100**
STRIKE 1	44	45	30	52	115
PAYOFF (SHORT CALL)	0	0	0	0	0
DELTA	0.9616	0.3950	0.0122	0.0014	0.9832
NUMBER OF CALLS	103.99	253.16	8190.75	71284.07	101.71
VALUE OF PORTFOLIO	4438.13	4485.72	2926.35	5081.12	11690.22
COST OF PORTFOLIO	4328.30	4506.58	2363.20	4369.20	112212.23
PROFIT	**109.84**	**-20.86**	**563.16**	**711.92**	**479.99**
PORTFOLIO PROFIT	**1844.04**				

Fig 10: Delta Hedging

TABLE XI

STOCKS	**AFL**	**APA**	**COG**	**MET**	**PRU**
NUMBER OF STOCKS	**100**	**100**	**100**	**100**	**100**
DELTA I	0.9619	0.395002	0.012209	0.001403	0.983218
DELTA II	0.00	0.94	0.79	0.31	0.45
GAMMA I	0.40	0.71	0.10	0.01	0.05
GAMMA II	0.03	0.21	0.90	0.91	0.45
NO. OF PUT I	-106.88	36.59	1336.17	37885.03	-106.86
NO. OF PUT II	1568.54	-121.50	-147.16	-492.11	11.26
VALUE OF PORTFOLIO	4438.30	4408.68	2928.18	5087.07	11693.62
COST OF PORTFOLIO	4248.61	4363.00	2308.18	4319.22	10816.03
PROFIT	**189.69**	**45.68**	**620.00**	**767.85**	**877.60**
PORTFOLIO PROFIT	**2500.81**				

Fig 11: Delta-Gamma Hedging

D. RISK ANALYSIS

The Investor

TABLE XII

DELTA-HEDGED (LONG POSITION)					
	DELTA	**GAMMA**	**VEGA**	**RHO**	**THETA**
STOCK	500	0	0	0	0
OPTION 1	-500	1442.6903	29088.0726	-14274.1	403.9282
TOTAL	0	1442.6903	29088.0726	-14274.1	403.9282
DELTA -GAMMA HEDGED					
	DELTA	**GAMMA**	**VEGA**	**RHO**	**THETA**
STOCK	500	0	0	0	0
OPTION 1	-308.9117	66.5072	-5284.3328	-5026.8129	286.2575
OPTION 2	-191.0883	-66.5072	-5284.3328	-9311.6402	421.9209
TOTAL	0	0	-2.36469E-11	-14338.45	708.1784

Fig 12: Risk Analysis for a Long Position

TABLE XIII

DELTA-HEDGED (SHORT POSITION)					
	DELTA	**GAMMA**	**VEGA**	**RHO**	**THETA**
STOCK	500	0	0	0	0
OPTION 1	-500	1874.74	20585.56	13988.56	-960.202
TOTAL	0	1874.74	20585.56	13988.56	-960.202
DELTA -GAMMA HEDGED					
	DELTA	**GAMMA**	**VEGA**	**RHO**	**THETA**
STOCK	500	0	0	0	0
OPTION 1	123.9308	556.307	-6838.071	-6314.97	228.6129
OPTION 2	376.0692	-556.307	-6838.071	-7414.09	459.3454
TOTAL	0	0	4.27E-11	-13929.1	687.9583

Fig 13: Risk Analysis for Short Position

CONCLUSION

This paper threw light on the price sensitivities of options and their uses as a hedging tool in financial markets. The Julia Programming language used, aided the computation of all the Greeks in a single line of code which simplified algorithmic differentiation. The results as shown in the previous chapter strongly indicate the importance of the Greeks in hedging a portfolio of stocks or options. It was observed that the delta-gamma hedged portfolio had a barely significant risk to volatility. Notwithstanding, the performance of the 9 selected stocks influenced the results obtained as a different portfolio could give unrealistic values.

RECOMMENDATION

Generally, a hedged portfolio is better than a holding a naked position as its risk is insignificant and as such less volatile. It will be beneficial to introduce derivatives market into the financial sector of the Ghanaian economy as it provides more alternatives for investment for individuals, corporate bodies and financial institutions.

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