A Forecasting Model Based on Particle Swarm Optimization and Weight Fuzzy Relation Matrix

Nghiem Van Tinh
Thai Nguyen University of Technology, Thai Nguyen University
Thai Nguyen, Vietnam

***Abstract*—Partitioning the universe of discourse and determining effective intervals are critical for forecasting in fuzzy time series. Equal length intervals used in most existing literatures are convenient but subjective to partition the universe of discourse. In this paper, a hybrid forecasting model based on two computational approaches, the weighted fuzzy relationship matrices and particle swarm optimization (PSO), is presented for academic enrolments. First, the weighted fuzzy relationship matrices are more effective to capture fuzzy relations on time series data than common fuzzy relationship. Second, the particle swarm optimization for the optimized lengths of intervals is developed to adjust interval lengths by searching the space of the universe of discourse. Then, based on the optimal intervals obtained, we fuzzify all the historical data of the enrolments of the University of Alabama and calculate the forecasted output by the proposed method. Compared to the other methods existing in literature on the historical data of the enrolments of the University of Alabama based on the first-order fuzzy time series and high - order fuzzy time series using. the proposed method gets a higher average forecasting accuracy rate than the existing methods.**

Keywords — Fuzzy time series (FTS), forecasting, fuzzy relationship matrix, PSO, enrolments.

I. INTRODUCTION

In the past years, fuzzy time series has been widely used for forecasting data of dynamic and non-linear data in nature. Many previous studies have been discussed for forecasting used fuzzy time series, such as crop forecast [6], [7] academic enrolments [1], [10], the temperature prediction [13], stock markets [14], etc. There is the matter of fact that the traditional forecasting methods cannot deal with the forecasting problems in which the historical data are represented by linguistic values. Ref. [1], [2] proposed the time-invariant FTS and the time-variant FTS model which use the max–min operations to forecast the enrolments of the University of Alabama. their methods performed max-min composition operations to handle the fuzzy process, which required a lot of computation time when a fuzzy relationship matrix (FRM) is large. Then, Ref. [3] proposed the first-order FTS model by introducing a more efficient arithmetic method. After that, FTS has been widely studied to improve the accuracy of forecasting in many applications. Ref. [4] considered the trend of the enrolment in the past years and presented another forecasting model based on the first-order FTS. He pointed out that the effective length of the intervals in the universe of discourse can affect the forecasting accuracy rate. In other words, the choice of the length of intervals can improve the forecasting results. Ref. [5] presented a heuristic model for fuzzy forecasting by integrating Chen's fuzzy forecasting method [3]. At the same time, Ref.[8] proposed several forecast models based on the high-order fuzzy time series to deal with the enrolments forecasting problem. In [9], the length of intervals for the FTS model was adjusted to forecast the Taiwan Stock Exchange (TAIEX). Ref. [11] present a new method for temperature prediction and the TAIFEX forecasting, based on high- order fuzzy logical relationships and genetic simulated annealing techniques. Recently, Ref.[16] presented a new hybrid forecasting model which combined particle swarm optimization with fuzzy time series to find proper length of each interval. Additionally, Ref.[17] proposed a new method to forecast enrolments based on automatic clustering algorithm and high – order fuzzy logical relationships.

In this paper, we proposed a hybrid forecasting model combining the time-invariant fuzzy relationship groups and automatic clustering technique in [18]. Recently, particle swarm optimization (PSO) has been successfully applied in many applications. Lai et al. [19] introduced a PSO algorithm to automatically determine the proper number of features to classify spam e-mails. Cui et al. [20] proposed a novel velocity threshold to improve the performance of PSO. Cui et al. [21] introduced chaotic sequences to expanding PDPSO for solving high-dimensional problems. Khalid et al. [22] proposed a model for PSO implementation to solve the DNA sequence design problem. Based on Chen's model [3], Kuo et al. [16] introduced a new hybrid forecasting model which combined fuzzy time series with PSO algorithm to find the proper length of each interval.

In case study, we applied the proposed model to forecast the enrolments of the University of Alabama.

Computational results show that the proposed model outperforms other existing methods.

Rest of this paper is organized as follows. The fundamental definitions of FTS and PSO algorithm are discussed in Section 2. In Section 3, we construct hybrid forecasting model which combined the weighted fuzzy relationship matrices and PSO algorithm. In Section 4 presents the results from the application of the proposed method for forecasting the enrolments of the University of Alabama. Finally, conclusions are presented in Section 5.

II. FUZZY TIME SERIES AND PSO ALGORITHM

In this section, we provide briefly some definitions of fuzzy time series [1], [2], [3] in Subsection A and PSO algorithm in Subsection B.

A. *Fuzzy Time Series*

In [1] , Song and Chissom proposed the definition of fuzzy time series based on fuzzy sets ,Let $U=\{u_1,u_2,\ldots,u_n\}$ be an universal set; a fuzzy set *A* of *U* is defined as $A=\{ f_A(u_1)/u_1+\ldots+f_A(u_n)/u_n\}$, where f_A is a membership function of a given set *A*, $f_A : U\rightarrow [0,1]$, $f_A(u_i)$ indicates the grade of membership of u_i in the fuzzy set A, $f_A(u_i) \in [0, 1]$, and $1\le i \le n$. General definitions of fuzzy time series are given as follows:

Definition 1: Fuzzy time series

Let *Y(t)* (*t* = ..., 0, 1, 2 …), a subset of R, be the universe of discourse on which fuzzy sets $f_i(t)$ (i = 1,2…) are defined and if *F(t)* be a collection of $f_i(t)$) (i = 1, 2…). Then, *F(t)* is called a fuzzy time series on *Y(t)* (t . . ., 0, 1,2, . . .).

Definition 3: Fuzzy relationship (FR)

If there exists a fuzzy relationship R(t-1,t), such that F(t) = F(t-1)$*$R(t-1,t), where "$*$" is an arithmetic operator, then F(t) is said to be caused by F(t-1). The relationship between F(t) and F(t-1) can be denoted by F(t-1)→ F(t). Let A_i = F(t) and A_j = F(t-1), the relationship between F(t) and F(t -1) is denoted by fuzzy logical relationship $A_i \rightarrow A_j$ where A_i and A_j refer to the current state or the left hand side and the next state or the right-hand side of fuzzy time series.

Definition 4: λ- order fuzzy time series

Let *F(t)* be a fuzzy time series. If F(t) is caused by F(t-1), F(t-2),…, F(t-λ+1) F(t-λ) then this fuzzy relationship is represented by by F(t-λ), …, F(t-2), F(t-1)→ F(t) and is called an λ- order fuzzy time series.

Hwang et al. [4] presented a new forecasting model in which fuzzy relations were represented as the fuzzy relationship matrices (FRMs). In Hwang's model, the fuzzy set *F*(*t*) was defined by the variation of the enrollments between years *t* and *t* - 1. In this paper, raw historical data are used to create the fuzzy sets. Therefore, the fuzzy set *F*(*t*) is defined by the enrollment of year *t*. A window base denotes how many past years data will be used in the FRMs.

Let *R*(*t*) be the FRM of a window base *m*, $M^m(t)$ be the operation matrix formed by the fuzzy sets of *w* past years, and *C*(*t*) be the criterion matrix formed by the fuzzy set of the current year. In the training phase, the matrices $M^m(t)$ and *C*(*t*) are defined as follows:

$$M^m(t) = \begin{bmatrix} F(t-1) \\ F(t-2) \\ \ldots\ldots. \\ F(t-m) \end{bmatrix} \quad (1)$$

$$\text{and } C(t) = F(t) \quad (2)$$

Based on equations (1) and (2), *R*(*t*) can be calculated as follows

$$R(t) = M^m(t) \otimes C(t) \quad (3)$$

where the symbol ⊗ denotes the product composition operation. By performing the transpose and max composition operations, *P*(*t*) can be derived from *R*(*t*) as follows:

$$P(t) = (\max(R(t)^T)^T \quad (4)$$

B. *Particle Swarm Optimization Algorithm*

PSO proposed by Dr. Eberhart and Dr. Kennedy, is a random searching algorithm based on group collaboration and is inspired by simulating the behavior of flock of birds. It is Particle swarm optimization initializes each particle randomly, and then finds the optimal solution through iteration. In each step, the particles update themselves by tracking their own best position and the best particle [16]. To get the optimal solution, the particles update their own speed and positions according to the following equation.

$$V_i^{k+1} = \omega^k * V_i^k + C_1 * Rand() * (P_{best} - X_i^k) + C_2 * Rand() * (G_{best} - X_i^k) \quad (5)$$

$$X_i^{k+1} = X_i^k + V_i^{k+1} \quad (6)$$

$$\omega^k = \omega_{max} - \frac{k*(\omega_{max}-\omega_{min})}{iter\_max} \quad (7)$$

where, the parameter ω denotes the initial weight coefficient, the symbols C_1 and C_2 are the self-confidence coefficient and the social confidence coefficient, respectively.

The pseudocode for PSO algorithm is summarized in the following algorithm 1

Algorithm 1: The pseudocode for PSO algorithm

1. initialize all particles' positions X_i and velocities V_i
2. **while** the stop condition (*the maximal moving steps are reached*) is not satisfied **do**

 2.1 **.for** particle i, ($1\le i \le$ NumberOfParticles) **do**

 calculate the fitness value of particle i (X_i)

 if fitness(X_i) > fitness($Pbest_i$) **then** $Pbest_i = X_i$ **end if**

 if fitness($Pbest_i$)> fitness($Gbest_i$) **then** $Gbest_i$ =$Pbest_i$

 end if

 end **for**

 2.2. **for** particle i, ($1\le i \le$ NumberOfParticles) **do**

 ✓ move particle i to another position according to (5) and (6)

end **for**

end **while**

return Gbest and corresponding position

III. Forecasting Model Based on Weighted Fuzzy Relation Matrices and PSO

An improved hybrid model based on Hwang's model [4] by combining the weighted FRMs and PSO algorithm is presented. First, raw historical data are used instead of the variations of historical data in our forecasting model. Second, the FRMs are derived from the corresponding FRs; and then the decreasing weighted scheme assigns the largest weights to the latest past fuzzy set of a FRM for capturing efficient fuzzy relations. Third, the PSO algorithm is developed to adjust the interval lengths to obtain the optimal partition. A detailed explanation of the proposed model follows.

A. *Forecasting model by the weighted FRMs.*

The step-wise procedure of the proposed forecasting model is detailed as follows:

Step 1: *Define the universe of discourse U*

Assume Y(t) be the historical data of enrolments at year t. The university of discourse is defined as $U = [D_{min}, D_{max}]$. In order to ensure the forecasting values bounded in the universe of discourse U, we set $D_{min} = I_{min} - N_1$ and $D_{max} = I_{max} + N_2$; where I_{min}, I_{max} are the minimum and maximum data of Y(t); N_1 and N_2 are two proper positive integers to tune the lower bound and upper bound of the U

Step 2:

Divide U into equal length intervals. Compared to the previous models in [3, 16], we cut U into seven intervals, $u_1, u_2, \dots, u_7$, respectively. The length of each interval is $l = \frac{D_{max}-D_{min}}{7}$. Thus, the seven intervals are: $u_1 = (D_{min}, D_{min}+l]$, $u_2 = (D_{min}+l, D_{min}+2l]$, …, $u_7 = (D_{min}+6l, D_{max}]$.

Step 3: Define the fuzzy sets for each interval

Assume that there are 7 intervals $u_1, u_1, u_1, \dots, u_7$ for data set obtained in Step 2. For 7 intervals, there are 7 linguistic values which are $A_1, A_2, A_3, \dots, A_6$ and A_n to represent different regions in the universe of discourse, respectively. Each linguistic variable represents a fuzzy set A_i $(1 \le i \le 7)$ and its definition is described in (8) .

$$A_i = \sum_{j=1}^{7} \frac{a_{ij}}{u_j} ; \quad (8)$$

where $a_{ij} \in [0,1]$, $1 \le i \le 7$, $1 \le j \le 7$ and u_j is the j-th interval. The value of a_{ij} indicates the grade of membership of u_j in the fuzzy set A_i and it is shown as following:

$$a_{ij} = \begin{cases} 1 & \text{if } j == i \\ 0.5 & \text{if } j == i-1 \text{ or } j == i+1 \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

Step 4: Fuzzify all historical data

In order to fuzzify all historical data, it's necessary to assign a corresponding linguistic value to each interval first. The simplest way is to assign the linguistic value with respect to the corresponding fuzzy set A_i that each interval u_j belongs to with the highest membership degree. The fuzzy set F(t) for Y (t) is expressed as F(t) = A_i.

Step 5: Identify all fuzzy relationships (FR)

Relationships are identified from the fuzzified historical data obtained in Step 4. If the fuzzified historical data of years t and t - 1 are A_i and A_j, respectively, then construct the first – order fuzzy logical relationship "$A_i \rightarrow A_j$", where A_i and A_j are called the fuzzy set on the left-hand side and fuzzy set on the right-hand side of fuzzy logical relationships, respectively.

Similarly for m-order fuzzy relationship, we should find out any relationship which has the $F(t-m), F(t-m+1), \dots, F(t-1) \rightarrow F(t)$, where $F(t-m), F(t-m+1), \dots, F(t-1)$ and $F(t)$ are called the current state and the next state, respectively. Then a m - order fuzzy relationship is got by replacing the corresponding linguistic values.

Step 6: Construct the weighted FRMs

The window base of a FRM is set to be *m* for a ^{m}th-order FR ($m \ge 1$). The current state and next state of a FR are converted to the operation matrix and the criterion matrix, respectively. The decreasing weighted scheme assigns the largest weights to the latest past fuzzy set of a FRM. Let $M^m(t)$ be an operation matrix, *C*(*t*) be a criterion matrix, $W^m(t)$be a weighted operation matrix, and *R*(*t*) be a FRM. The operation matrix is obtained by the fuzzy sets of the current state as follows

$$M^m(t) = \begin{bmatrix} F(t-1) \\ F(t-2) \\ \dots\dots \\ F(t-m) \end{bmatrix} = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ a_{21} & \dots & a_{2n} \\ \vdots & & \vdots \\ a_{m1} & \dots & a_{mn} \end{bmatrix} \quad (10)$$

where a_{ij} refers to the membership value of the fuzzy set F(t - i), $1 \le i \le m$ and $1 \le j \le n$. Based on the fuzzy sets of an operation matrix, the weighted operation matrix is calculated by the DW scheme which assigns the weights from w to 1 decreasingly. In the case of Eq.(12), we assign m, m-1,… and 1 to fuzzy sets A_{i1}, A_{i2},.. and A_{im}, respectively. Thus, the weighted operation matrix derived forming $W^m(t)$ is given as

$$W^m(t) = \begin{bmatrix} a_{11} * m & \dots & a_{1n} * m \\ a_{21} * (m-1) & \dots & a_{2n} * (m-1) \\ \vdots & & \vdots \\ a_{m1} * 1 & \dots & a_{mn} * 1 \end{bmatrix} \quad (11)$$

The criterion matrix is defined by the fuzzy set of the next state as follows

$$C(t) = (F(t) = [c_1 \quad c_2 \quad \cdots \quad c_n] \quad (12)$$

where c_j refers to the membership value of the interval u_j in the fuzzy set *F*(*t*).

The weighted FRM is computed by the product composition operation of the weighted operation matrix and the criterion matrix. For example, the weighted FRM in the training phase is calculated as follows

$$R(t) = W^m(t) \otimes C(t) \quad (13)$$

Step 7: Calculate the forecasted outputs.

Use the transpose and max composition operations in the weighted FRM as shown in Eq.(13). The raw membership of the forecasted output *F'*(*t*) is obtained as follows:

$$P(t) = [\, p_1 \;\; p_2 \;\; \dots \;\; p_n] = (\max(R(t)^T)^T \quad (14)$$

Then, the raw membership of the forecasted output is normalized as follows

$$F(t) = [\, f_1 \;\; f_2 \;\; \dots \;\; f_n] = [\frac{p_1}{\sum_{i=1}^{n} p_i} \; \frac{p_2}{\sum_{i=1}^{n} p_i} \cdots \frac{p_n}{\sum_{i=1}^{n} p_i}] \quad (15)$$

The final forecasted value is calculated by the product of the normalized membership matrix in Eq.(18) as follows:

$$\text{Forecasted\_value} = f_1 * m_1 + f_1 * m_2 + \ldots + f_n * m_n \quad (16)$$

Where, m_i is the midpoint matrix of interval u_i.

Calculate forecasting accuracy by using Mean Square Error (MSE) according to Eq.(17).

$$MSE = \frac{1}{n}\sum_{i=m}^{n} (f_i - r_i)^2 \quad (17)$$

Where, r_i denotes actual data at year i, f_i is forecasted value at year i, n is number of the forecasted data, m is the order of the fuzzy relationships.

B. *Interval partition by PSO algorithm.*

The PSO algorithm is used to minimize the MSE value (17) by adjusting the interval lengths. In proposed model, each particle exploits the intervals in the universe of discourse of historical data Y(t). Let the number of the intervals be n, the lower bound and the upper bound of the universe of discourse on historical data Y(t) be p_0 and p_n, respectively. Each particle is a vector consisting of n-1 elements p_i where $1 \le i \le n-1$ and $p_i \le p_{i+1}$. Based on these n-1 elements, define the n intervals as $u_1 = [\, p_0, p_1]$, $u_2 = [p_1, p_2]$,..., $u_i = [p_{i-1}, p_i]$,…, $u_{n-1} = [\, p_{n-2}, p_{n-1}]$ and $u_n = [\, p_{n-1}, p_n]$, respectively. When a particle moves to a new position, the elements of the corresponding new vector need to be sorted to ensure that each element p_i ($1 \le i \le n-1$) arranges in an ascending order. The complete steps of the proposed method are presented in Algorithm 2.

***Algorithm 2*: Interval partition by PSO**

1. initialize all particles' positions X_i and velocities V_i

2. **while** the stop condition (maximum iterations or minimum MSE criteria) is not satisfied **do**

2.1**.** **for** particle i, ($1 \le i \le$ NumberOfParticles) **do**

- ✓ Define fuzzy sets based on the current position of particle i by Step 3
- ✓ Fuzzify all historical data by Step 4
- ✓ Establish all m^{th}-order fuzzy relationships by Step 5
- ✓ Construct all m^{th}-order weighted fuzzy relationship matrix by Step 6
- ✓ Calculate forecasting values by Step 7
- ✓ Compute the MSE values for particle i based on (17)
- ✓ Update the personal best position of particle i according to the MSE values mentioned above.

end for

2.2. Update the global best position of all particles according to the MSE values mentioned above.

3. **for** particle i, ($1 \le i \le$ NumberOfParticles) **do**

- ✓ move particle i to another position according to (5) and (6)

end for

- ✓ update ω according to (7)

end while

IV. Experimental Results

A. *Experimental results for forecasting enrollments*

In this paper, we apply the proposed model to forecast the enrolments of University of Alabama with the whole historical data [3] from the period 1971 to 1992 are used to perform comparative study in the training phase. Without loss of generality, the standard PSO model with the same experimental parameters as the model [16] is used. The essential parameters of proposed model for forecasting enrolment are listed in Table 1.

Table I: Parameters used the proposed forecasting model

Number of particles	30
Maximum number of iterations	100
The value of inertial weigh **ω** be linearly decreased	1.4 to 0.4
The coefficient $C_1 = C_2$	2
The velocity V_i be limited to	[-100,100]
The position X_i be limited to	[13000,20000]

To estimate the performance of the proposed model with different number of intervals, five forecasting models in SCI [1], C96 [3], H01H [12], CC06a [10] and HPSO [16] are selected for comparison. All forecasting models use the first-order FRs with different number of intervals. A comparison of all forecasting results is listed in Table 2. To be clearly visualized, Fig.1 depicts the trends for actual data and forecasted results between the proposed model and its counterparts. From Fig.1, it can see that the predicted values and the actual values are very close and the forecasting accuracy by the MSE value of the proposed model is more precise than among two compared models with different order fuzzy logical relationship The smallest MSE value of 16227 is obtained from the proposed method. It is obvious that the forecasting accuracy of the proposed model is more precise than any existing models for the first-order FRs with different number of intervals.

TABLE II: A COMPARISON OF THE FORECASTED RESULTS OF PROPOSED MODEL WITH THE EXISTING MODELS BASED ON THE FIRST-ORDER FUZZY TIME SERIES UNDER DIFFERENT NUMBER OF INTERVALS.

Year	Actual data	SCI [1]	C96 [3]	H01 [12]	CC06a [10]	HPSO [16]	Proposed model
1971	13055	-	-	-	-	-	-
1972	13563	14000	14000	14000	13714	13555	13548
1973	13867	14000	14000	14000	13714	13994	13849
1974	14696	14000	14000	14000	14880	14711	14680
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1990	19328	19000	19000	19000	19300	19340	19345
1991	19337	19000	19000	19500	19149	19340	19318
1992	18876	19000	19000	19149	19014	19014	18836
MSE		**423027**	**407507**	**226611**	**35324**	**22965**	**16227**

To verify the forecasting effectiveness for high-order FLRs, C02 [8], CC06b [11], and HPSO [16] models are used to compare with the proposed model. Table 3 states that the proposed model has smaller MSE values than any of the models presented in articles [8,11,16]. The proposed model also gets the lowest MSE value of 16017 for the 7th-order FRMs among all the compared models. For easily visualizing, form Fig. 2, it can clearly show that the forecasting accuracy of the proposed model is more precise than those of existing models with different high-order FRMs. The average MSE value of the proposed model is 17565.9, which is smallest among all forecasting models compared

TABLE III: A COMPARISON OF THE FORECASTED ACCURACY BETWEEN PROPOSED MODEL AND ITS COUNTERPARTS WITH DIFFERENT NUMBER OF ODERS.

Methods	Number of oders								
	2	3	4	5	6	7	8	9	Average(*MSE*)
C02 model	89093	86694	89376	94539	98215	104056	102179	102789	**95868**
CC06b model	67834	31123	32009	24948	26980	26969	22387	18734	**31373**
HPSO model	67123	31644	23271	23534	23671	20651	17106	17971	**28121**
Our model	22302	18144	17040	16642	16437	16017	16931	17014	**17565.9**

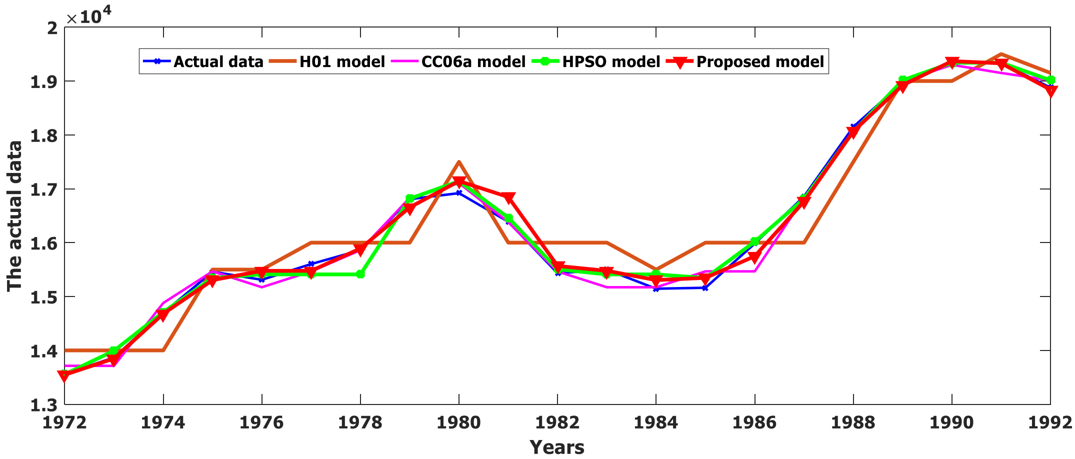

Fig. 1: The curves of the proposed model and its counterparts based on first – order weighted FRMs

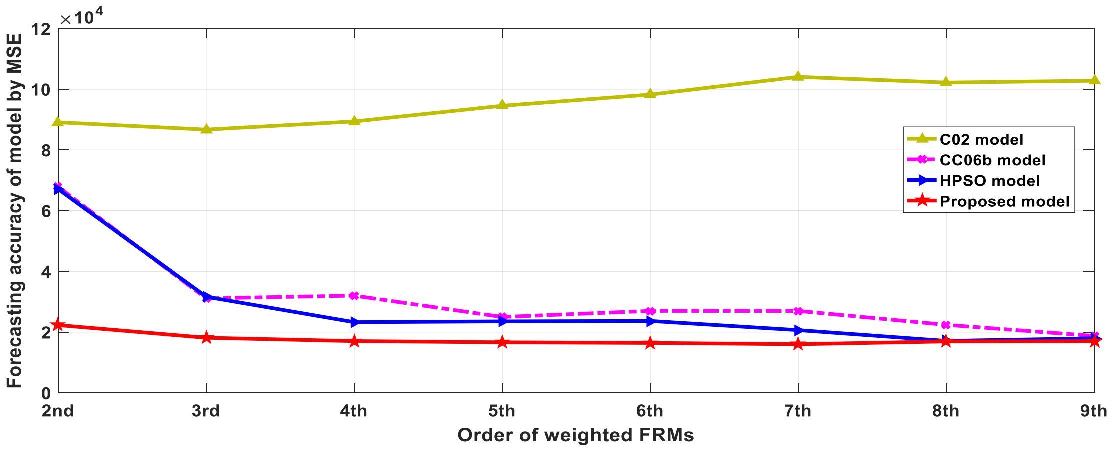

Fig. 2: A comparison of the MSE values based on the high-order FTS model with different intervals

V. Conclusions

In this paper, we have presented a hybrid forecasted method to handle forecasting enrolments of the University of Alabama based on the weighted fuzzy relationship matrices and PSO algorithm. Firstly, the FRMs with a decreasing weight scheme are more effective to capture fuzzy relations on time series data than FLR rules does. Secondly, the PSO algorithm for the optimized lengths of intervals is developed to adjust the interval lengths by searching the space of the universe of discourse. Thirdly, we calculate forecasting output and compare forecasting accuracy with other existing models. Lastly, based on the performance comparison in Tables 2 & 3 and Figs. 1& 2, it can show that our model outperforms previous forecasting models for the training phases with various orders and different interval lengths.

The proposed model was only tested by the forecasting enrollment problem, it can actually be applied to other practical problems such as earthquake forecast, and weather prediction in the further research.

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